Networks

Data Sheet

Networks are used to model many real-life situations. For example, the network below is a simple map showing estimated distances and times between some English towns.

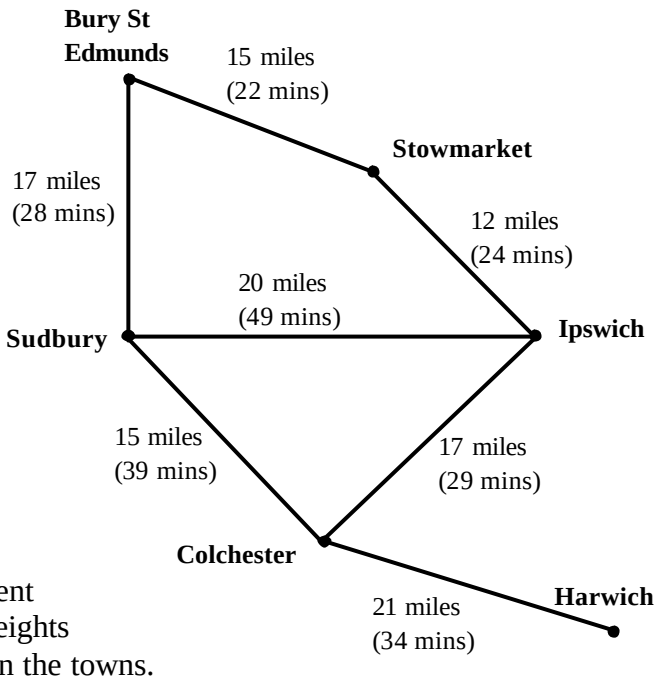
Some definitions involving networks are given below:

A **graph** is a diagram consisting of **edges** and **vertices** that represent how objects are related to each other.

A **vertex** (or node) is a point where edges meet. The vertex is **even** or **odd** according to whether an even or odd number of edges meet there.

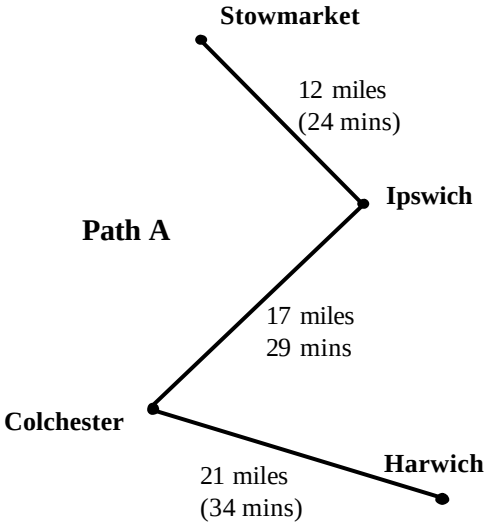
An **edge** is a line joining two vertices. It can be **directed** (i.e. one-way) or **undirected** (two-way).

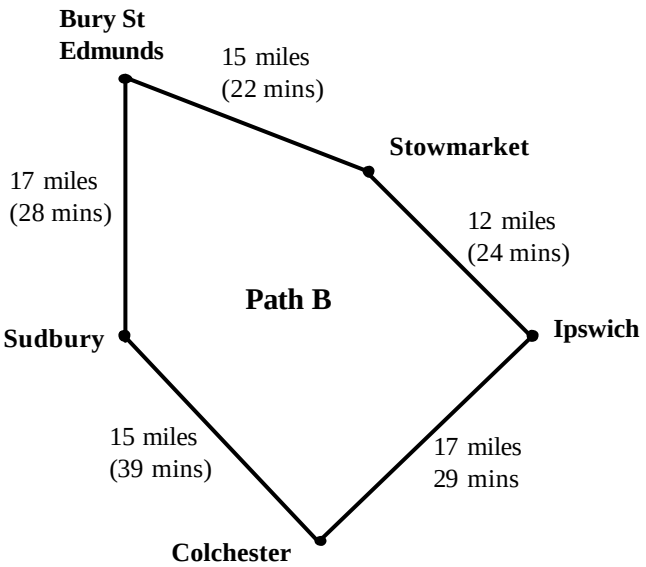
A **weight** can be allocated to an edge. This may represent distance, time or costs.



In the graph shown here, the vertices represent towns and the edges represent roads with weights representing the distances (or times) between the towns.

A **path** is a route through the graph that does not visit any vertex more than once and does not go along any edge more than once. The graph above includes lots of paths. Two examples are given below.

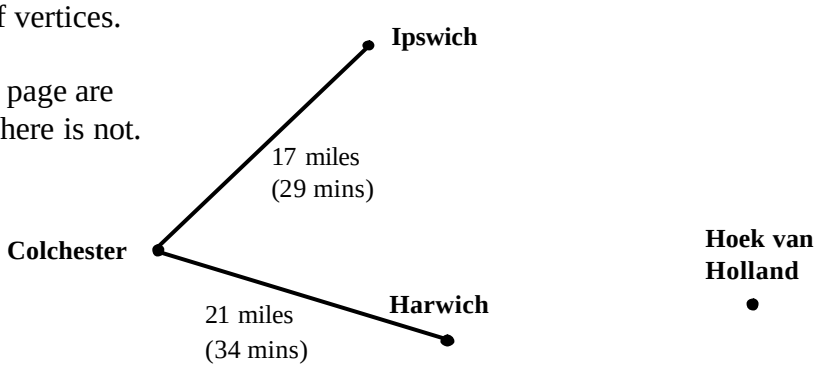




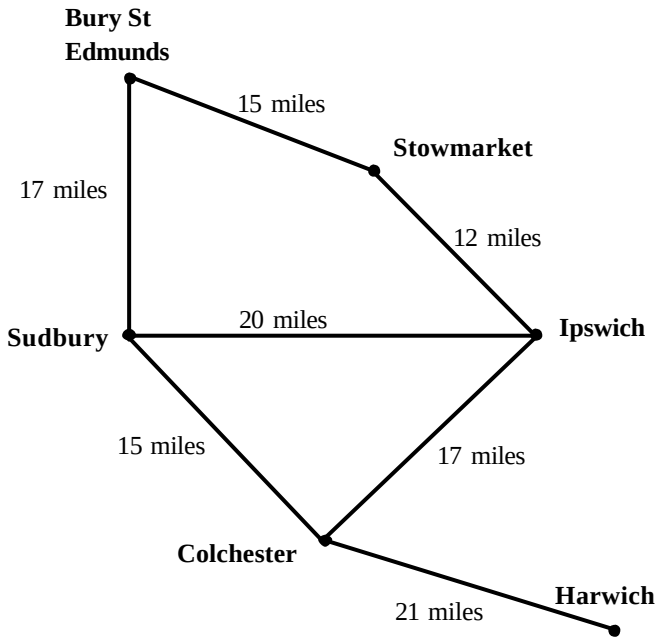
A **cycle** is a path that forms a loop by returning to its starting point.
Path B above is a cycle, but Path A is not a cycle.

A graph is **connected** if there is at least one route between each pair of vertices.

All the graphs on the previous page are connected, but the one shown here is not. There is no road joining any of the English towns to Hoek van Holland (though you can get there by ferry from Harwich).



A **matrix** is an array of numbers. In an **adjacency matrix** these numbers represent the number of edges that directly join each pair of vertices. In a **distance matrix** the numbers give the distance between each pair of vertices. The graph of the road network from the previous page and its adjacency and distance matrices are given below.



Note
In the matrices below:

B	represents	Bury St Edmunds
Su	represents	Sudbury
C	represents	Colchester
H	represents	Harwich
I	represents	Ipswich
St	represents	Stowmarket.

Adjacency matrix

	B	Su	C	H	I	St
B	0	1	0	0	0	1
Su	1	0	1	0	1	0
C	0	1	0	1	1	0
H	0	0	1	0	0	0
I	0	1	1	0	0	1
St	1	0	0	0	1	0

Distance matrix

	B	Su	C	H	I	St
B	0	17	0	0	0	15
Su	17	0	15	0	20	0
C	0	15	0	21	17	0
H	0	0	21	0	0	0
I	0	20	17	0	0	12
St	15	0	0	0	12	0

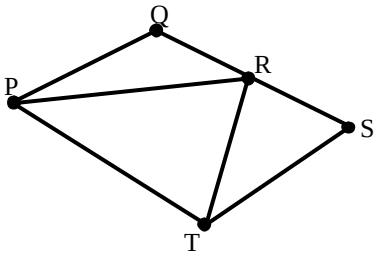
Networks

Worksheet

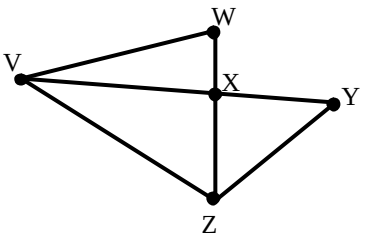
1 Copy and complete the table for the graphs shown below it.

Graph	Number of edges	Even nodes	Odd nodes
A			
B			
C			

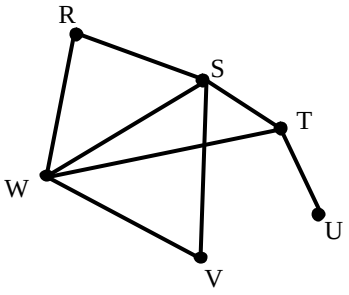
Graph A



Graph B



Graph C (Note the intersection of SV and WT is not a node)



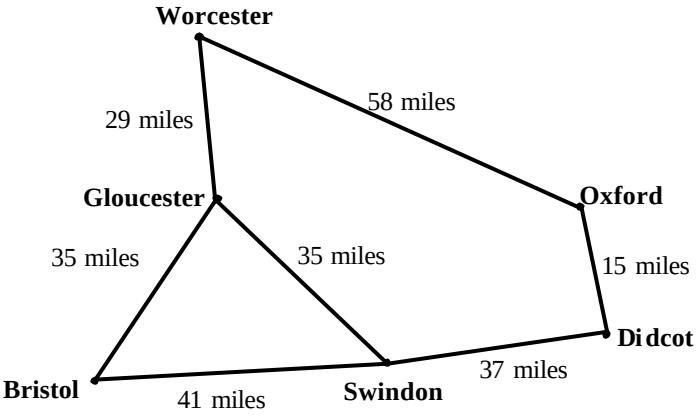
2 The graph shows the rail network connecting six places in England.

a) Draw an adjacency matrix.

b) Draw a distance matrix.

c) Sketch two paths through the graph.

d) List nodes that give a cycle in the graph.



3 This distance matrix gives the mileages between Manchester (M), Leeds (L), Sheffield (S), Doncaster (D) and Kingston upon Hull (K).

Draw the corresponding graph of the network. Show on it the distances given in the matrix.

	M	L	S	D	K
M	0	40	38	0	0
L	40	0	33	29	55
S	38	33	0	18	0
D	0	29	18	0	47
K	0	55	0	47	0

4 a) Draw a sketch of the road network joining towns in your local area.
Maps and distances (and estimated times) can be found using the AA Route Planner at www.theaa.com/travelwatch/planner_main.jsp .

b) Draw the corresponding adjacency and distance matrices.

Teacher Notes

Unit Advanced Level, Using and applying decision mathematics

Notes on Activity
Pages 1 and 2 introduce some of the terms used when working with networks. Many of the definitions and examples are also included in the Powerpoint presentation of the same name - this can be used to introduce this subject. The worksheet on Page 3 gives some short questions that can be used to check whether learners can remember what is meant by some of the terms as well as a final question that requires use of the internet to find information from which learners can draw a local road network diagram.

Answers to Worksheet

1

Graph	Number of edges	Even nodes	Odd nodes
A	7	Q, R, S	P, T
B	7	W, X, Y	V, Z
C	8	R, S, V, W	T, U

2 a) **Adjacency matrix**

	B	G	W	O	D	S
B	0	1	0	0	0	1
G	1	0	1	0	0	1
W	0	1	0	1	0	0
O	0	0	1	0	1	0
D	0	0	0	1	0	1
S	1	1	0	0	1	0

b) **Distance matrix**

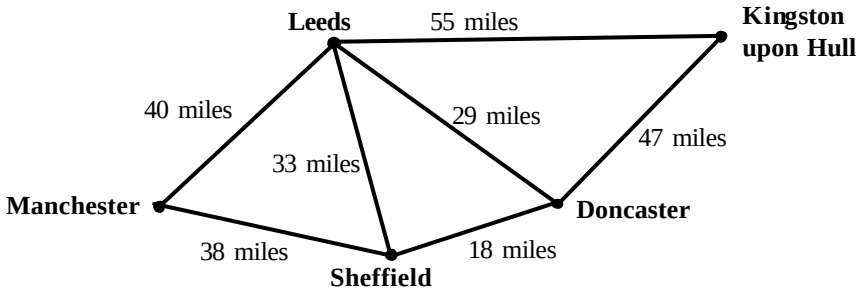
	B	G	W	O	D	S
B	0	35	0	0	0	41
G	35	0	29	0	0	35
W	0	29	0	58	0	0
O	0	0	58	0	15	0
D	0	0	0	15	0	37
S	41	35	0	0	37	0

where
B is Bristol
G is Gloucester
W is Worcester
O is Oxford
D is Didcot
S is Swindon

c) Sketches of any two routes that do not visit any vertex more than once and do not go along any edge more than once (eg BGWO, GBSDO)

d) Any list of nodes that give a path that forms a loop by returning to its starting point (eg GBSG, GSDOWG, GBSDOWG).

3



4 a) Sketch of road network joining local towns.
b) Corresponding adjacency and distance matrices.