Coastal Erosion

Coastal erosion is the process by which cliffs and rocks are broken up by the action of the sea and transported out to sea or distributed along the coast by waves and the wind.

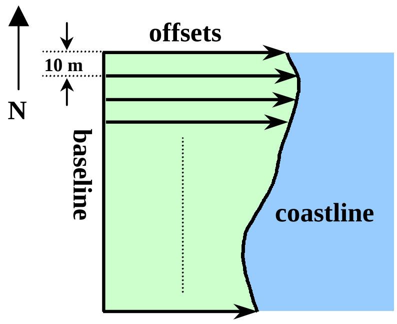
The Holderness Coastline of Yorkshire suffers from the worst coastal erosion in Europe. During the last 2000 years the coastline has retreated by almost 400 metres and over 30 villages between Bridlington and Spurn Head have been lost to the sea. Sea defences have reduced erosion at important points along the coastline, but it is not possible to protect every part of the coast.

The erosion at unprotected points occurs mainly during storms and tidal surges. The average rate of erosion is about 2 metres per year.

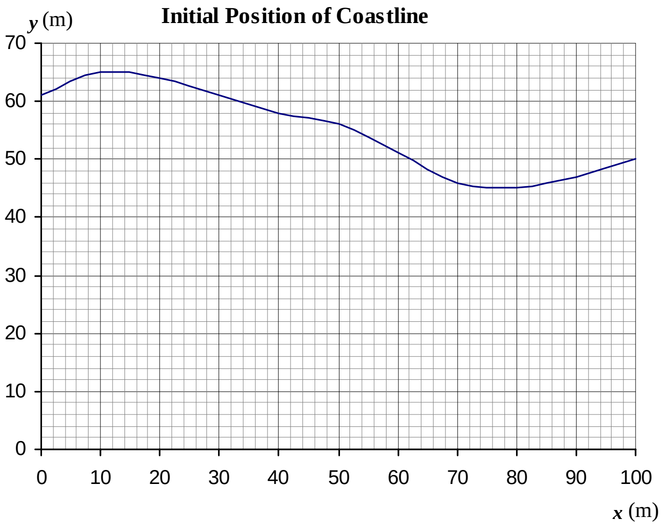
Land Area

The area of a piece of land with an irregular coastline can be estimated using a variety of methods.

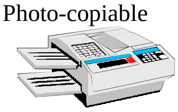
Suppose we wish to estimate the area of the piece of the land shown in the sketch and estimate how much of this land will be lost by coastal erosion over a 10 year period.



The initial position of the coastline can be defined by measuring a series of offsets from a baseline as shown. In this case the baseline has been taken to be a line drawn from North to South and the offsets have been taken at intervals of 10 metres.
The baseline can then be taken as the *x* axis as shown below.
The table gives the offsets as *y* co-ordinates taken at 10 metre intervals of *x*.



x (metres)	***y*** (metres)
0	61
10	65
20	64
30	61
40	58
50	56
60	51
70	46
80	45
90	47
100	50

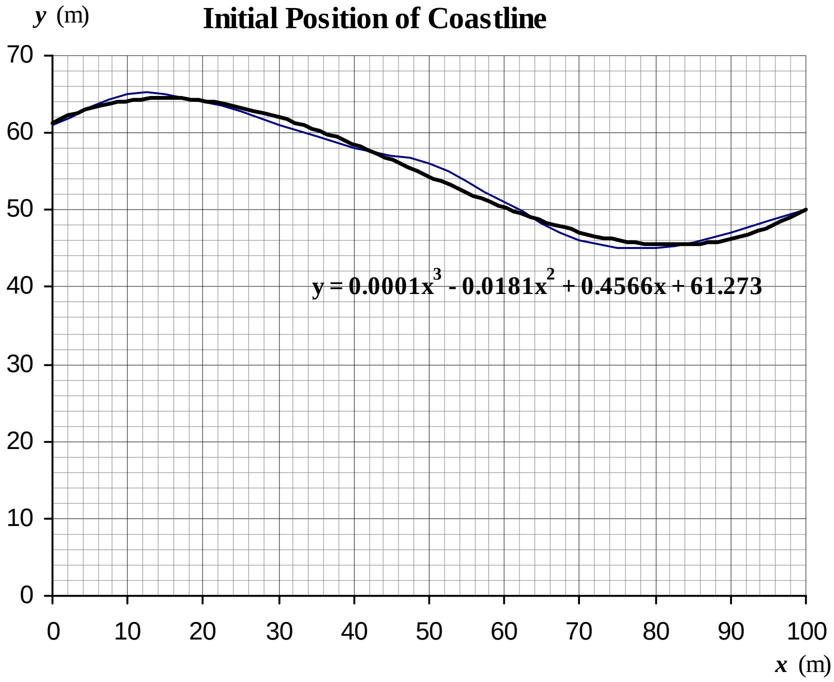


Using Integration

The coastline can be modelled by a cubic function.

The cubic function shown superimposed on the original graph was found using a trendline in Excel. If you have a graphic calculator, use it to find a cubic model. It should be similar to this, but may not be exactly the same.

The area of land between the base line and the coastline is approximately equal to the integral of the cubic function between 0 and 100.



Area

$$\approx \int_0^{100} \left(0.0001x^3 - 0.0181x^2 + 0.4566x + 61.273\right)dx$$

$$= \left[0.000025x^4 - 0.0060\dot{3}x^3 + 0.2283x^2 + 61.273x\right]_0^{100}$$

$$= 2500 - 6033 + 2283 + 6127 \ = 4877 \text{ m}^2$$

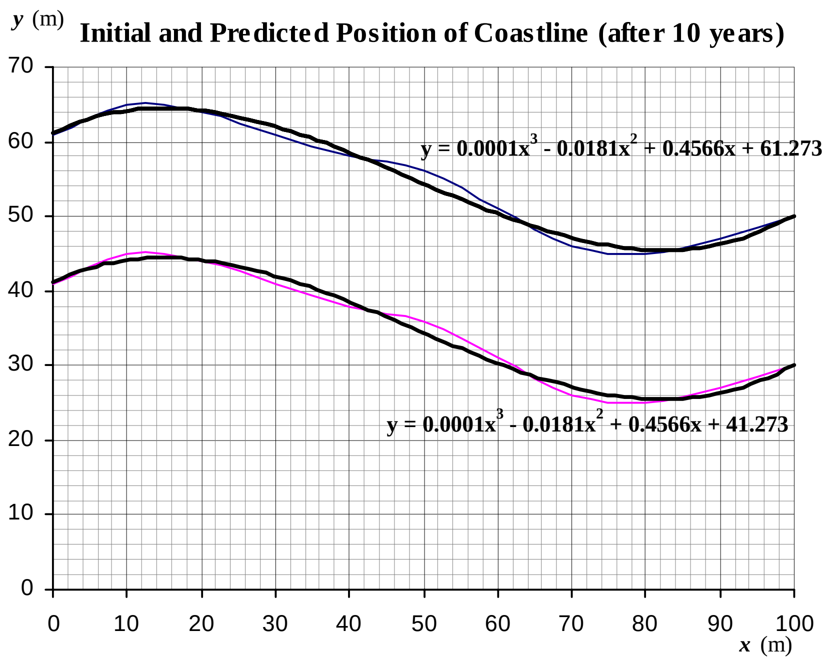
If you have found your own cubic model, use integration to find an estimate of the area and compare your result with the estimate given here.

The initial area of the land is approximately 4900 m^2

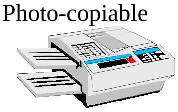
If we assume that the coastline erodes at a rate of 2 metres per year, then in 10 years it will recede by approximately 20 metres.

This graph shows both the initial position of the coastline and its predicted position after 10 years.

The cubic function that models the new position of the coastline is almost identical to that which models its initial position. The only difference is that the constant term is 20 smaller, reflecting the reduction of all *y* co-ordinates by 20.



When integration is used to find the area of land remaining after 10 years, the majority of the working is the same as before.



Remaining Area $\approx \int_0^{100} \left(0.0001x^3 - 0.0181x^2 + 0.4566x + 41.273\right)dx$

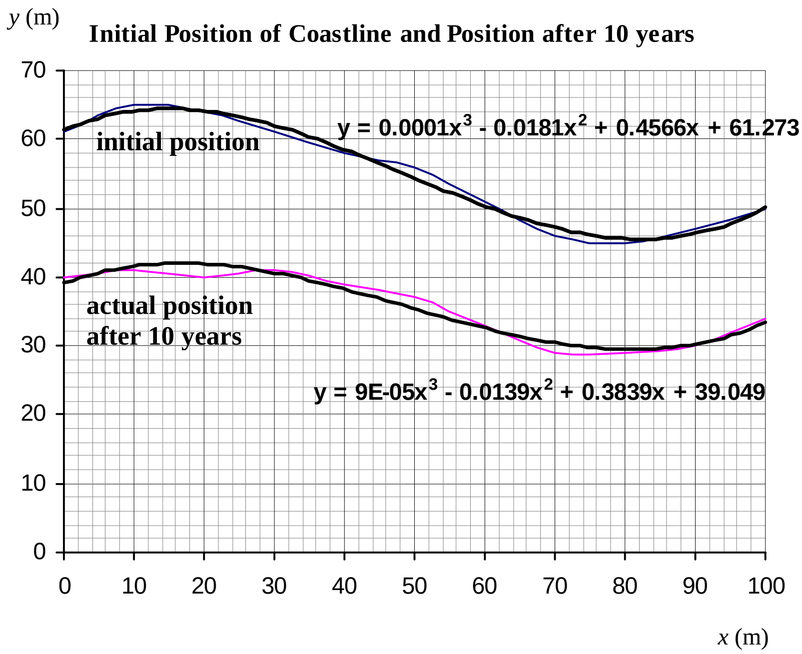
$$= \left[0.000025x^4 - 0.0060\dot{3}x^3 + 0.2283x^2 + 41.273x\right]_0^{100}$$

$$= 2500 - 6033 + 2283 + 4127 = 2877 \text{ m}^2\text{, approximately } 2900 \text{ m}^2$$

The loss of land is approximately 4877 – 2877 i.e **2000 m^2**
Note that this value is equal to the length of the baseline (100 metres) multiplied by the reduction in the lengths of the offsets (20 metres). Can you explain this using the graph?

In practice the coastline will recede more at some points than others. Suppose the actual offsets from the baseline after 10 years are as shown in the table below. The initial position of the coastline and its actual position after 10 years are shown on the graph together with the cubic models given by Excel.

x (m)	*y* (m) initial	y_a (m) after 10 yrs
0	61	40
10	65	41
20	64	40
30	61	41
40	58	39
50	56	37
60	51	33
70	46	29
80	45	29
90	47	30
100	50	34



Using the cubic model given for the new position of the coastline:

Remaining Area $\approx \int_0^{100} \left(0.00009x^3 - 0.0139x^2 + 0.3839x + 39.049\right)dx$

$$= \left[0.0000225x^4 - 0.0046\dot{3}x^3 + 0.19195x^2 + 39.049x\right]_0^{100}$$

$$= 2250 - 4633 + 1920 + 3905 = 3442 \text{ m}^2\text{, approximately } 3400 \text{ m}^2$$

The loss of land is approximately 4877 – 3442 =1435 i.e **1400 m^2** (to 2 sf)

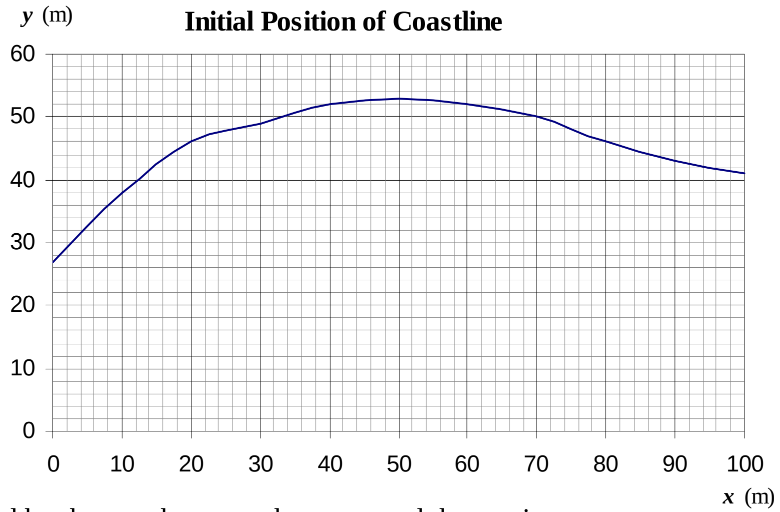
If you have a graphic calculator use it to find a cubic model for the position of the coastline after 10 years. Integrate the function to estimate the remaining area and the loss of land. Compare your results with the estimates given above.

In the work that follows use quadratic or cubic models to represent the position of the coastlines given.

Try these…

1 The coordinates give the position of a coastline, with *y* denoting the lengths of offsets taken at intervals of 10 metres.

x (m)	***y*** (m)
0	27
10	38
20	46
30	49
40	52
50	53
60	52
70	50
80	46
90	43
100	41

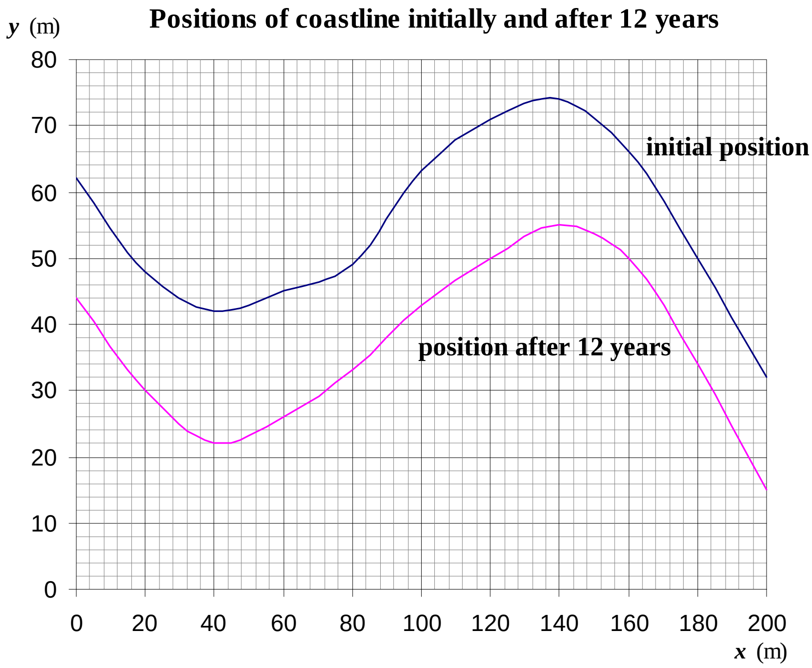


a Find the area of land represented by the area between the curve and the *x* axis.

b If the coastline recedes at an average rate of 1.8 metres per year, estimate the area of the land lost to the sea in a period of 10 years.

2 The table and graph give the initial position of a coastline and its position 12 years later.

x (m)	***y*** (m) initial	y_a(m) after 12 years
0	62	44
20	48	30
40	42	22
60	45	26
80	49	33
100	63	43
120	71	50
140	74	55
160	66	50
180	50	34
200	32	15

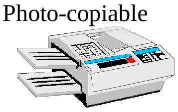


Estimate the area of land lost.

3 The table below gives the lengths of offsets from a baseline to a coastline taken at 50 metre intervals:

Distance along baseline (m)	0	50	100	150	200	250	300	350
Initial offset (m)	76	62	54	46	40	36	34	32
Offset after 20 years (m)	39	33	28	24	20	18	16	15

Estimate the area of land that has been eroded in the 20 year interval.



Teacher Notes

Unit Advanced Level, Modelling with calculus

Skills used in this activity:
- use of Excel or graphic calculator to find quadratic and cubic functions to model curves
- definite integration of quadratic and cubic functions

Notes on Activity

This activity uses integration to estimate the area of land and land loss due to coastal erosion.
Points for discussion may include:
- methods for finding quadratic and cubic functions to model curves
- how well the models fit the curves
- comparisons of the answers given using different models (see answers for questions 1 and 3 below)

In another activity called Coastal Erosion A, the same example and exercise are used but the answers are evaluated using the trapezium rule rather than integration. Each activity can be used by itself or the two activities can be combined allowing comparisons to be made between the methods. If you wish to use the two activities together you will need to provide each student with a photocopy of pages 2 and 3 from this activity in addition to pages 1 – 4 of Coastal Erosion A.

The coastlines given in questions 1 and 3 of the exercise can be modelled by either quadratic functions or cubic functions. The answers below give both. It is expected that cubic functions are used in question 2. All answers were calculated using a graphic calculator. Answers given by Excel or a different type of graphic calculator may be different.

Answers

1 Quadratic Model of initial position: $y = 29.71 + 0.8366x - 0.007529x^2$

a 4644 = 4600 m^2 (to 2 sf) b 1800 m^2

Cubic Model of initial position: $y = 0.00007129x^3 - 0.01822x^2 + 1.244x + 27.14$

a 4643 = 4600 m^2 (to 2 sf) b 1800 m^2

2 Cubic model of initial position: $y = 61.83 - 1.026x + 0.01604x^2 - 0.00005863x^3$

Cubic Model of position after 12 years: $y = 43.92 - 1.056x + 0.01623x^2 - 0.00005863x^3$

Loss of area ≈ 11 170 – 7492 = 3678 = 3700 m^2 (to 2 sf)

3 Quadratic Model of initial position: $y = 0.0003619x^2 - 0.2476x + 75$

Quadratic Model of position after 20 years: $y = 0.0001643x^2 - 0.1258x + 38.96$ giving:

Loss of area ≈ 16 257 – 8275 = 7982 = 8000 m^2 (to 2 sf)

Cubic Model of initial position: $y = 75.64 - 0.2810x + 0.0006165x^2 - 0.0000004848x^3$

Cubic Model of position after 20 years: $y = 38.98 - 0.1272x + 0.0001749x^2 - 0.0000000202x^3$

Loss of area ≈ 16 255 – 8276 = 7979 = 8000 m^2 (to 2 sf)

