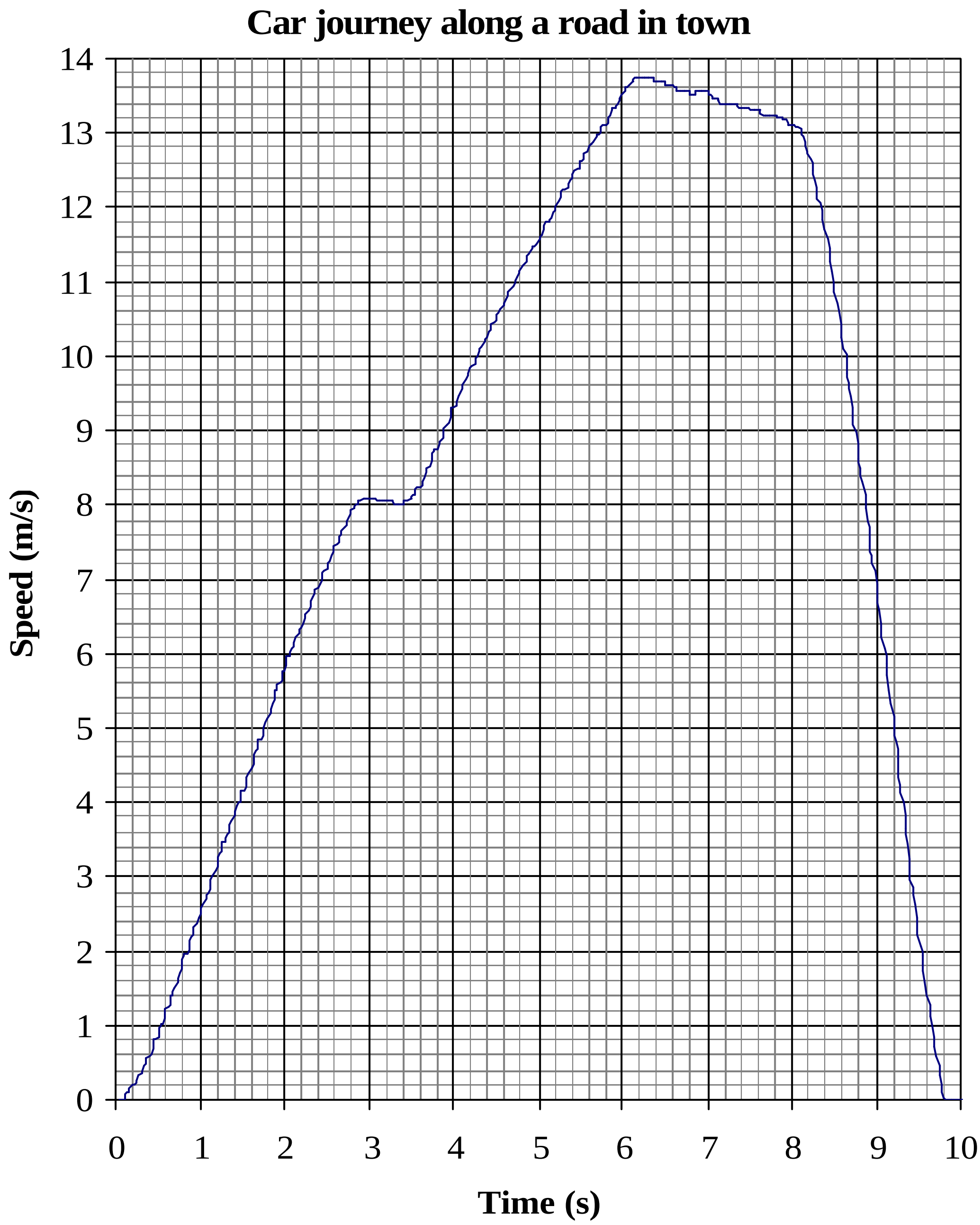
# Mean Values

The graph shows data collected from an accelerometer as a car travels along a road in town.




Describe what you think happens during this short journey.

How could you find the car's average (mean) speed?

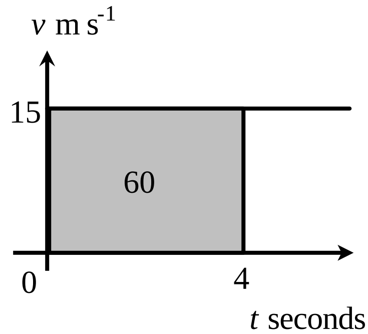
## Average speed

$$\textbf{Mean speed} = \frac{\textbf{Total distance travelled}}{\textbf{Total time taken}}$$

where

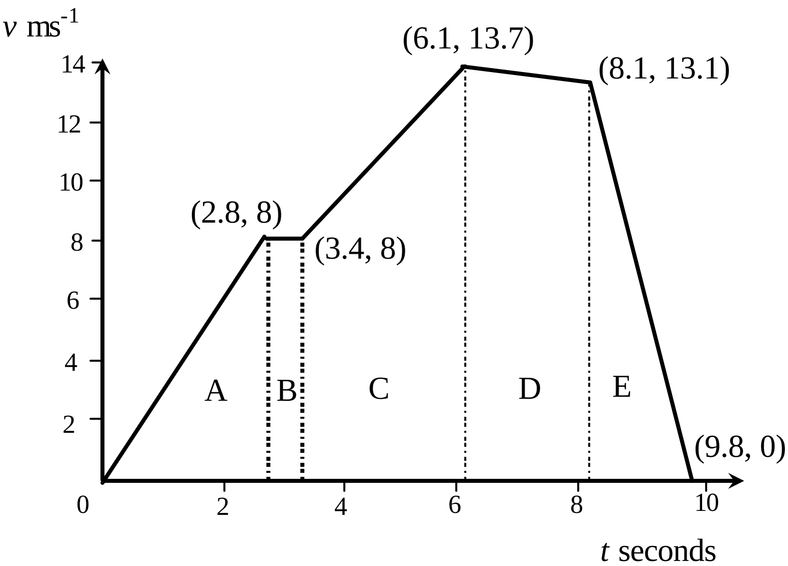
**Distance travelled = Area under the speed-time graph**

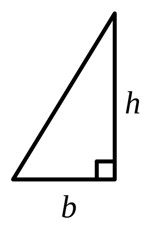
For example, if a car travels at a steady speed of 15 m s$^{-1}$, the distance travelled in 4 seconds is $4 \times 15 = 60$ m.



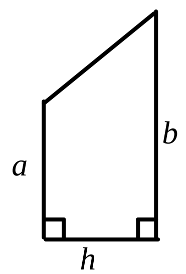

**Area of a rectangle = *l* ´ *b***

For the car travelling along the road in town, the graph can be modelled by a series of straight lines and the area under the graph can be estimated using triangles, rectangles and trapezia:




**Area of a triangle =** $\dfrac{b \times h}{2}$




**Area of a trapezium =** $\dfrac{(a+b)\times h}{2}$

Complete the following:

Area of A = ……………………………………………….

Area of B = ……………………………………………….

Area of C = ……………………………………………….

Area of D = ……………………………………………….

Area of E = ……………………………………………….

Total area = Total distance travelled = …………………..

**Average speed = ………………………………………..**





## Finding areas by integration

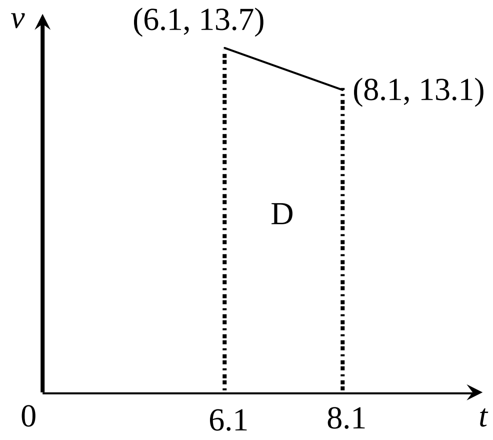
You can estimate the area under a graph by integrating a function that models the graph.

**Example**

For section D of the graph on page 2

The line joining (6.1, 13.7) and (8.1, 13.1) has gradient:

$$m = \frac{13.1 - 13.7}{8.1 - 6.1} = \frac{-0.6}{2} = -0.3$$




Substituting (6.1, 13.7) in $y = mx + c$ gives:

$$13.7 = -0.3 \times 6.1 + c$$

giving $c = 13.7 + 0.3 \times 6.1 = 15.53$

The equation of the line is $y = 15.53 - 0.3x$

i.e. $v = 15.53 - 0.3t$

Using integration:

$$\text{Area of D} = \int_{6.1}^{8.1} (15.53 - 0.3t)\, dt = \left[15.53t - \frac{0.3t^2}{2}\right]_{6.1}^{8.1} = \left[15.53t - 0.15t^2\right]_{6.1}^{8.1}$$

$$= \left[15.53 \times 8.1 - 0.15 \times 8.1^2\right] - \left[15.53 \times 6.1 - 0.15 \times 6.1^2\right]$$

$$= 115.9515 - 89.1515 = \mathbf{26.8}$$

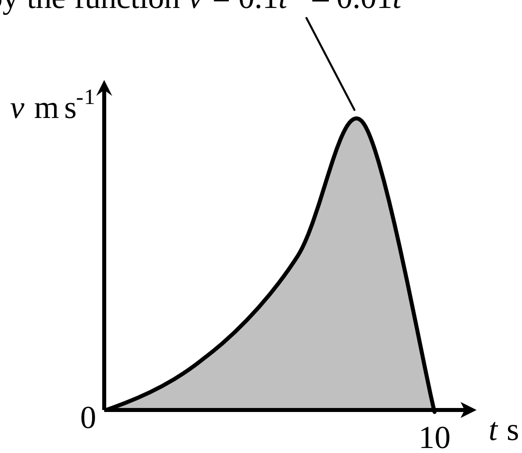
- Use integration to find the areas of the other sections shown in the graph on page 2.
- Compare the answers with the values you found earlier.
- Which method do you prefer and why?

**Example** – Using integration to find the area under a curve.

The speed of a cyclist along a road can be modelled by the function $v = 0.1t^3 - 0.01t^4$

$$\text{Area} = \int_0^{10} \left(0.1t^3 - 0.01t^4\right) dt = \left[\frac{0.1t^4}{4} - \frac{0.01t^5}{5}\right]_0^{10}$$

$$= \left[0.025t^4 - 0.002t^5\right]_0^{10} = 250 - 200 = 50$$




Distance travelled = 50 metres.

$$\text{Average speed} = \frac{50}{10} = \mathbf{5\ m\ s^{-1}}$$

1. For each part:
   - use a graphic calculator or spreadsheet to draw a speed-time graph
   - describe what happens during the time interval
   - use integration to find the area under the curve and hence estimate the average speed.

   a) The speed of a car, $v$ m s$^{-1}$, as it travels down the sliproad onto a motorway can be modelled by $v = 10 + 0.6t^2 – 0.04t^3$ for $0 = t = 10$ where $t$ is the time in seconds.

   b) The speed of a motorbike , $v$ m s$^{-1}$, as it comes to a halt at a T-junction can be modelled by $v = 0.096t^3 – 1.08t^2 + 18.2$ where $t$ is the time in seconds.

The mean values of other variables can also be found using integration. Try these:

2 The value of shares in a company over a five year period is modelled by

$$y = 0.0072x^3 – 0.51x^2 + 8.8x + 180$$

where $y$ is the value in pence and $x$ is the time in months.

a) (i) Use a graphic calculator or spreadsheet to draw the graph of this function for $0 = x = 60$.

(ii) Describe what happened to the value of the shares over this 5 year period.

b) The mean value of the shares over this period is given by

$$\text{Mean value} = \frac{\int_0^{60}\left(0.0072x^3 - 0.51x^2 + 8.8x + 180\right)dx}{60}$$

Find this mean value, giving your answer to the nearest pence.

3 The depth of water at Sunderland docks over a 12 hour period can be modelled by

$$y = 5.4 + 0.6x – 0.702x^2 + 0.1x^3 – 0.0038x^4$$

where $y$ is the depth in metres and $x$ is the time in hours after 3am.

a) (i) Use a graphic calculator or spreadsheet to draw the graph of this function for $0 = x = 12$.

(ii) According to your graph, when was low tide and what was the depth of the water at this time?

b) Find the mean depth of water during this period using

$$\text{Mean depth} = \frac{\int_0^{12}\left(5.4 + 0.6x - 0.702x^2 + 0.1x^3 - 0.0038x^4\right)dx}{12}$$

## Teacher Notes

**Unit** Advanced Level, Modelling with calculus

**Skills used in this activity**
- using areas under graphs to estimate mean values

**Preparation**
Students need to know how to
- find the area of rectangles, triangles and trapezia
- integrate polynomial functions.

**Notes on Activity**
By working through pages 1 – 3 of this activity students will learn how to use areas under speed-time graphs to estimate distance travelled and find mean speeds. This includes finding areas using triangles, rectangles and trapezia as well as by integration. The main points are also included in the Powerpoint presentation of the same name. This can be used to introduce the topic and illustrate some of the working.
Page 4 of the activity gives more practice in finding mean values and includes other real-life contexts.
If your students have calculators that can integrate, they could use them to check their answers.

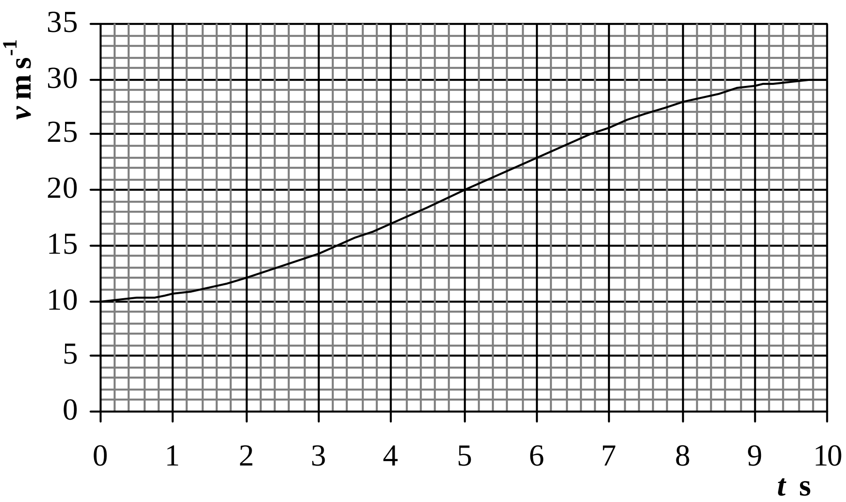
**Answers**

Pages 2 & 3
A = 11.2 B = 4.8 C = 29.295 D = 26.8 E = 11.135
Total distance = 83.23 m Average speed = 8.5 m $s^{-1}$ (to 1 dp)

Page 4

1 a)

**Speed-time graph for car travelling down sliproad**

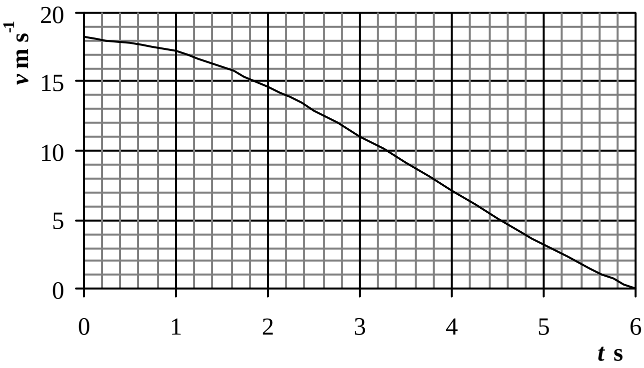



The car accelerates from 10 m $s^{-1}$ to 30 m $s^{-1}$
Area under curve = 200 Average speed = 20 m $s^{-1}$

b)

**Speed-time graph for motorbike coming to a halt**



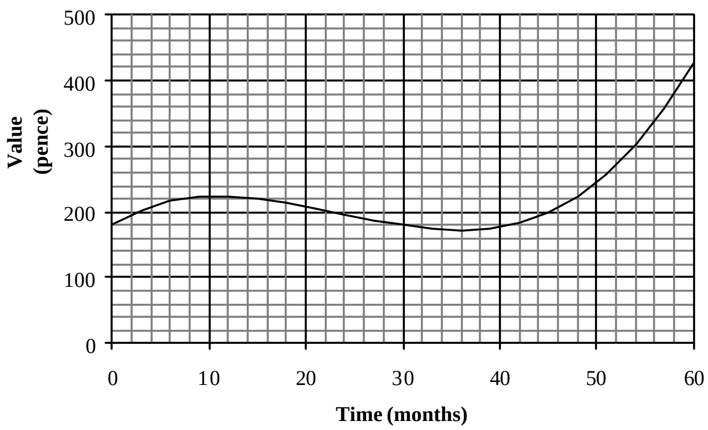

The motorbike decelerates from 18.2 m $s^{-1}$ to 0 m $s^{-1}$ in approximately 6 seconds.

Area under curve = 62.5

Average speed = 10.4 m $s^{-1}$ (1dp)

2 a)(i)

**Value of shares over 5 year period**

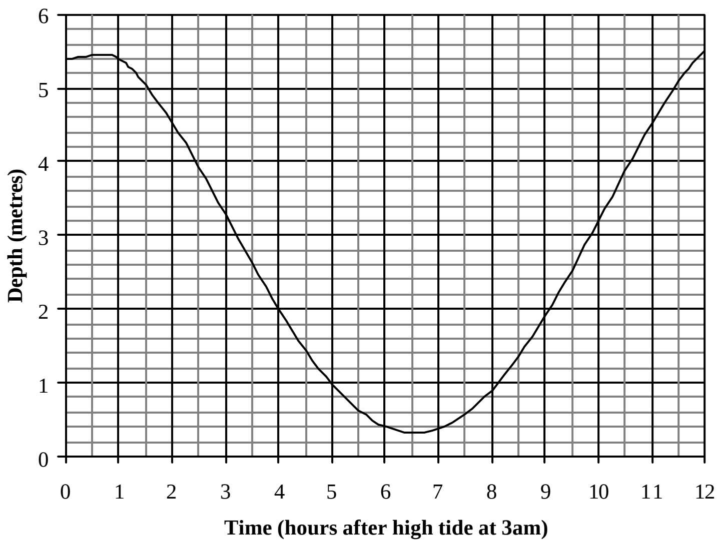



(ii) The value of the shares increases from 180 p to 225 p after 11 months, then falls to approximately 175 p after 36 months. The value then rises sharply to 427 p at the end of the 5 year period.

(b) 221 p (nearest pence)

3 a) (i)

**Water Depth at Sunderland docks**




(ii) Approximately 9:30 am with a depth of approximately 0.3 m

(b) 2.7 metres (to 1dp)

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