What's it worth?

Most cars depreciate in value as they get older.

There are a variety of ways in which this situation can be modelled mathematically.

Some suggestions are given below:

Model A The value of the car depreciates at a constant rate.
Model B The rate of depreciation is proportional to the age of the car.
Model C The rate of depreciation is proportional to the value of the car.
Model D The rate of depreciation is inversely proportional to the age of the car.
Model E The rate of depreciation is inversely proportional to the square root of the age of the car.
Model F The rate of depreciation is inversely proportional to the value of the car.
Model G The rate of depreciation is proportional to the square of the value of the car.
Model H The rate of depreciation is proportional to the square root of the value of the car.

Which of these models do you think are likely to be good models of the situation?
Investigate one or more of the models in depth.

As part of the investigation you should:

- collect some real data about the value of a car at different ages;
- find the value of a car as a function of age for the model(s);
- compare predictions from the model(s) with the real data you have collected;
- make use of the key features of functions and their graphs (eg gradients, intercepts) in reaching your conclusions;

If you have studied more than one model:

- say which model you prefer and why;
- explain your preference by comparing the features of these model(s) with those of models you have rejected.

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Teacher Notes

Unit Advanced Level, Modelling with calculus

Skills required in this actvity:

- solving simple differential equations involving powers, exponential and logarithmic functions;
- sketching graphs;
- interpreting key features of functions and graphs (eg gradients, intercepts, asymptotes) and comparing these to what happens in a real situation.

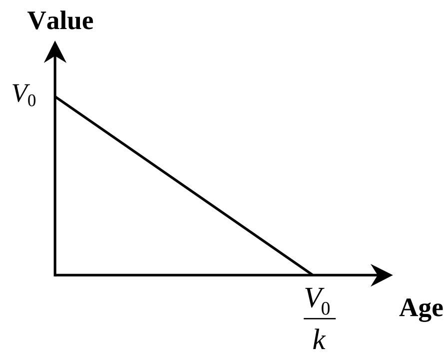
Notes on Activity

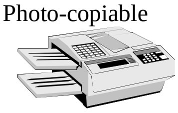
You will need to decide how many models you want students to study and whether students should work individually or in groups.
The differential equation, its solution and a sketch graph are given below for each of the models. In all cases V represents the value of the car at age t years. V_0 denotes the initial value of the car and k is a constant.
There are several trade guides on the market that give information about how the prices of cars fall with time. Some data is given in the Excel spreadsheet, What's it worth.xls, but please note that this data is likely to go out-of-date very quickly and you may need to update it before use.
It is expected that most students will use the real data to evaluate V_0 and k for a particular car (or cars) and hence find particular solutions, but more able students could be encouraged to find more general solutions like those given below and then use the data they collect to evaluate these solutions.
It would be useful to conclude the activity with a class discussion in which students present and discuss their findings.

Model A **Value of car depreciates at a constant rate**

Differential equation: $\frac{dV}{dt} = -k$

Solution: $V = V_0 - kt$





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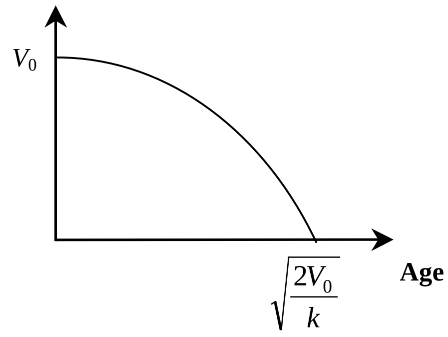
Model B Rate of depreciation is proportional to age of the car

Differential equation: $\frac{dV}{dt} = -kt$

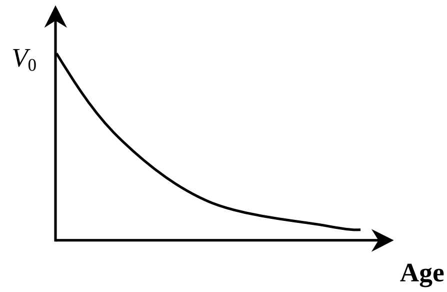
Solution: $V = V_0 - \frac{1}{2}kt^2$



Model C Rate of depreciation is proportional to value of the car

Differential equation: $\frac{dV}{dt} = -kV$

Solution: $V = V_0 e^{-kt}$



Model D Rate of depreciation is inversely proportional to age of car

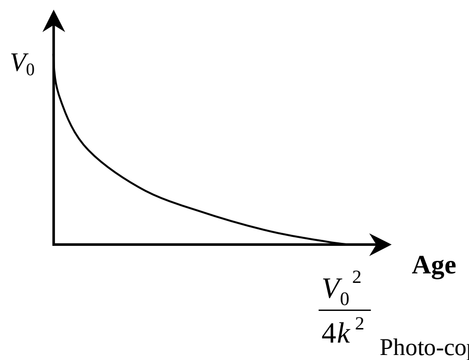
Differential equation: $\frac{dV}{dt} = -\frac{k}{t}$

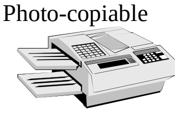
Solution: $V = -k\ln t + c$ is unsuitable because of discontinuity at $t = 0$

Model E Rate of depreciation inversely proportional to square root of age of car

Differential equation: $\frac{dV}{dt} = -\frac{k}{\sqrt{t}}$

Solution: $V = V_0 - 2k\sqrt{t}$

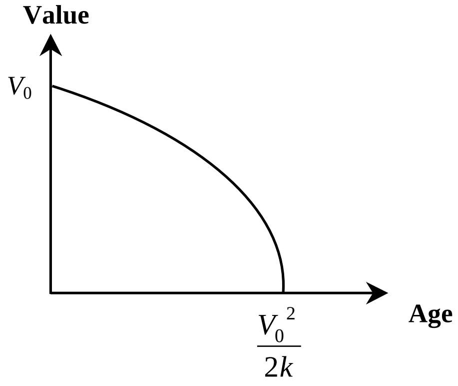




Model F Rate of depreciation is inversely proportional to value of car

Differential equation: $\frac{\mathrm{d}V}{\mathrm{d}t} = -\frac{k}{V}$

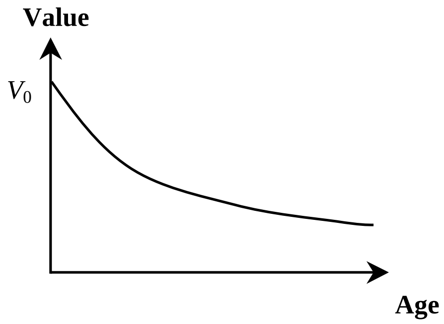
Solution: $V = \sqrt{V_0^{\ 2} - 2kt}$



Model G Rate of depreciation is proportional to square root of value of car

Differential equation: $\frac{\mathrm{d}V}{\mathrm{d}t} = -kV^2$

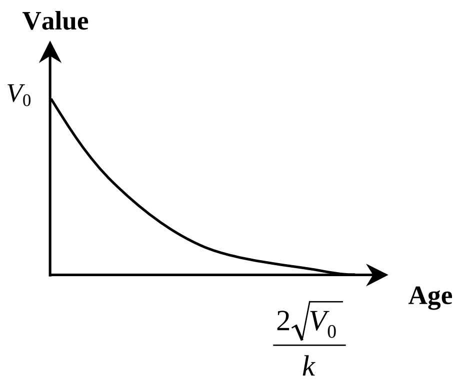
Solution: $V = \frac{V_0}{1 + kV_0 t}$



Model H Rate of depreciation is proportional to square root of value of car

Differential equation: $\frac{\mathrm{d}V}{\mathrm{d}t} = -k\sqrt{V}$

Solution: $V = \left(\sqrt{V_0} - \tfrac{1}{2}kt\right)^2$



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