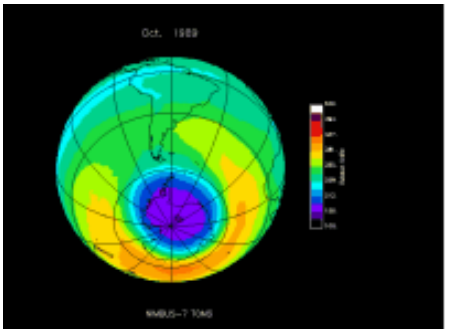
The Ozone Hole

Data Sheet

At ground level ozone is a health hazard, but the layer of ozone in the stratosphere (between 10 kilometres and 40 kilometres above the earth's surface) provides a valuable shield against harmful ultraviolet radiation from the sun and is crucial for life on earth. It is a matter of great concern that a significant depletion of the ozone layer has been detected in recent years



Source of Image: NASA website at:
http://toms.gsfc.nasa.gov/ozone/ozone.html

Ozone levels are monitored from the ground, air and space. Ozone levels vary periodically with the seasons and ozone depletion is most significant in an area over the Antarctic where levels of ozone fall so low in spring that scientists describe the loss as the 'Ozone Hole'.

The amount of ozone above a point on the earth's surface is measured in Dobson Units (DU). This measurement represents the thickness of the ozone layer if all the ozone molecules that are overhead could be brought down to the earth's surface.

It has been found that the average ozone layer thickness is about 300 DU which is equivalent to 3 mm. There is said to be an 'ozone hole' where the thickness is less than 220 DU.

The table shows the minimum thickness of the ozone layer and the average area of the Antarctic ozone hole between September 5th and October 25th for most years from 1980 to 2003. These measurements were taken from NASA satellites. These and other data are available from the US Environmental Protection Agency's website at:
http://www.epa.gov/ozone/science/hole/sizedata.html

Year	Minimum Ozone Level (Dobson Units)	Average Area of Ozone Hole (million km^2)
1980	205	0.456
1981	205	0.177
1982	189	3.138
1983	169	7.554
1984	154	9.868
1985	146	14.137
1986	159	11.072
1987	120	19.747
1988	173	8.556
1989	124	19.422
1990	128	18.127
1991	117	18.906
1992	124	21.937
1993	94	24.017
1994	88	23.429
1995	No satellite in place.	
1996	111	22.662
1997	104	21.465
1998	90	25.440
1999	92	22.529
2000	94	26.408
2001	99	25.082
2002	140	Not available
2003	97	Not available

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Worksheet

Listed below are functions suggested as possible models for the data given on the Data Sheet.

In each case t represents the number of years since 1980, so $t = 0$ represents the year 1980, $t = 1$ represents the year 1981 and so on.

Minimum Ozone Level

Linear Model A: $y = 190 - 5.3t$

Quadratic Model B: $y = 0.28t^2 - 11t + 205$

Exponential Model C: $y = 195e^{-0.04t}$

Average Size of Ozone Hole

Linear Model X: $y = 1.2t + 4$

Quadratic Model Y: $y = 2.6t - 0.065t^2$

Exponential Model Z: $y = 28\left(1 - e^{-0.1t}\right)$

For each model investigate:

- how well the model fits the data given on the Data Sheet
- what the model predicts for future years.

Teacher Notes

Unit Advanced Level, Working with algebraic and graphical techniques

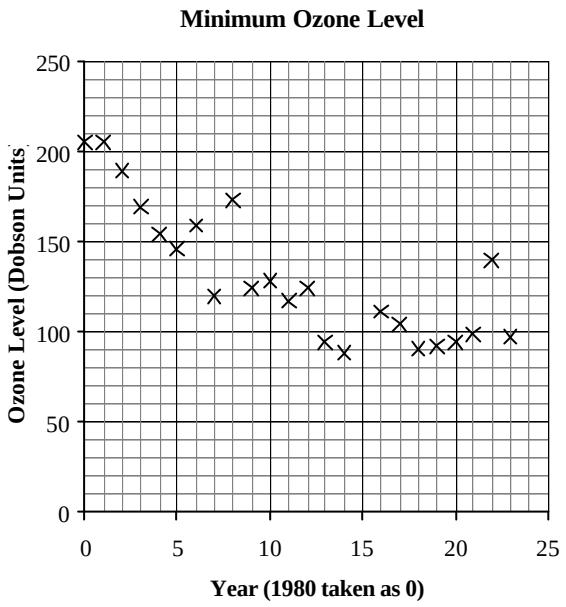
Notes
There are a variety of ways in which you could use this activity. If used relatively early in the course, students may need a lot of help - you could start by discussing ways in which students could tackle the task, perhaps working through the first set of models with them. Or if the activity is used near the end of the course, you might expect students to decide what to do themselves.

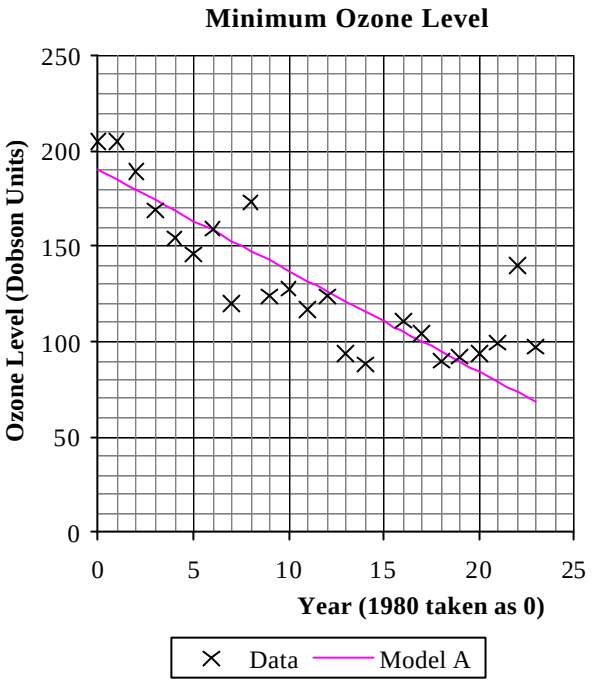
The models and real data can be compared using percentage errors as well as graphs drawn on graph paper or using graphic calculators or computers. The data has been supplied on an Excel spreadsheet to save time if computers are used. In this case students could investigate all the models quickly and easily. If you decide to use graph paper you could split the work between groups of students and pool results.

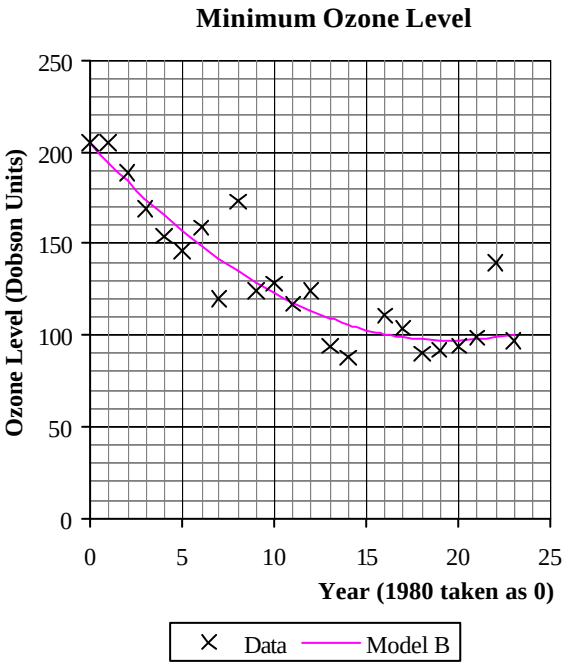
Percentage errors and graphs for each model are given below:

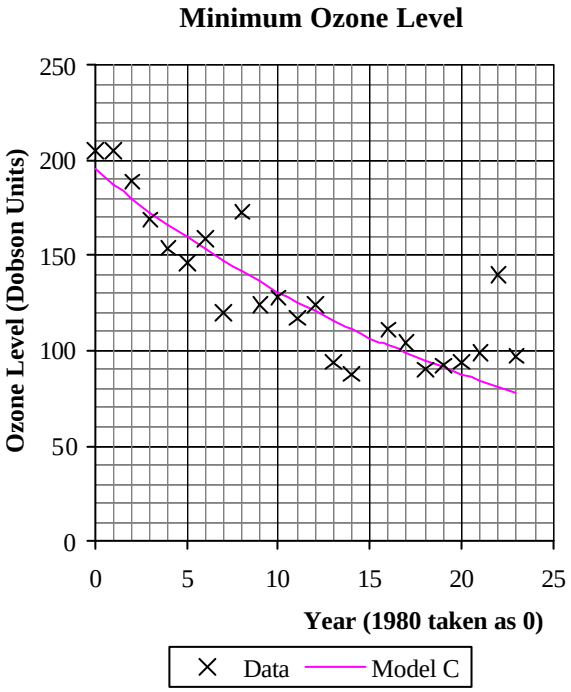
Minimum Ozone Level

t	Minimum Ozone (Dobson Units)	Model A	Model B	Model C	Model A % Error	Model B % Error	Model C % Error
0	205	190.0	205.0	195.0	-7.3	0.0	-4.9
1	205	184.7	194.3	187.4	-9.9	-5.2	-8.6
2	189	179.4	184.1	180.0	-5.1	-2.6	-4.8
3	169	174.1	174.5	172.9	3.0	3.3	2.3
4	154	168.8	165.5	166.2	9.6	7.5	7.9
5	146	163.5	157.0	159.7	12.0	7.5	9.4
6	159	158.2	149.1	153.4	-0.5	-6.2	-3.5
7	120	152.9	141.7	147.4	27.4	18.1	22.8
8	173	147.6	134.9	141.6	-14.7	-22.0	-18.2
9	124	142.3	128.7	136.0	14.8	3.8	9.7
10	128	137.0	123.0	130.7	7.0	-3.9	2.1
11	117	131.7	117.9	125.6	12.6	0.8	7.3
12	124	126.4	113.3	120.7	1.9	-8.6	-2.7
13	94	121.1	109.3	115.9	28.8	16.3	23.3
14	88	115.8	105.9	111.4	31.6	20.3	26.6
16	111	105.2	100.7	102.8	-5.2	-9.3	-7.4
17	104	99.9	98.9	98.8	-3.9	-4.9	-5.0
18	90	94.6	97.7	94.9	5.1	8.6	5.5
19	92	89.3	97.1	91.2	-2.9	5.5	-0.9
20	94	84.0	97.0	87.6	-10.6	3.2	-6.8
21	99	78.7	97.5	84.2	-20.5	-1.5	-15.0
22	140	73.4	98.5	80.9	-47.6	-29.6	-42.2
23	97	68.1	100.1	77.7	-29.8	3.2	-19.9



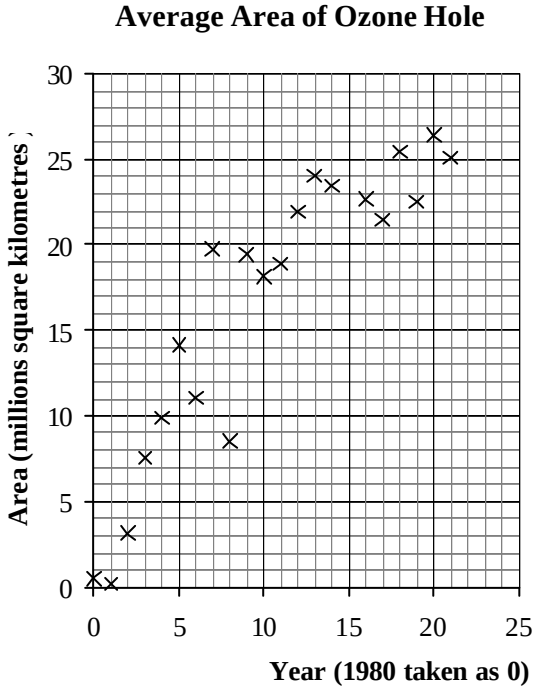


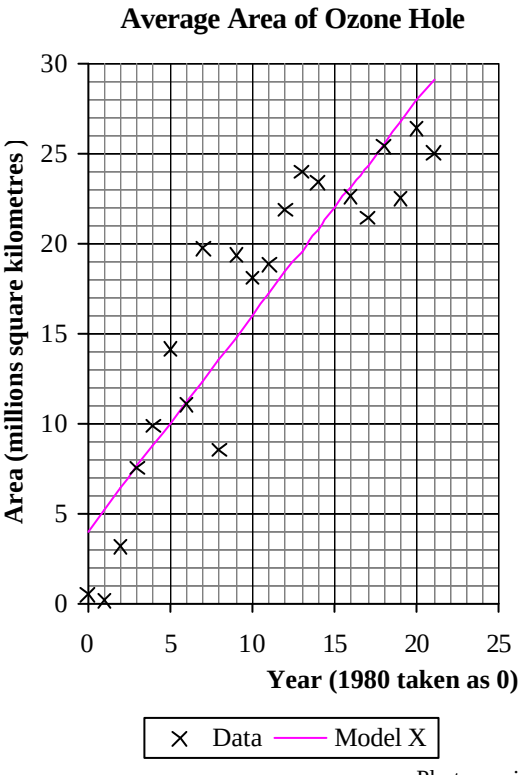




Average Area of Ozone Hole

t	Maximum Area (million km^2)	Model X	Model Y	Model Z	Model X % Error	Model Y % Error	Model Z % Error
0	0.456	4.0	0.0	0.0	777.2	-100.0	-100.0
1	0.177	5.2	2.5	2.7	2837.9	1332.2	1405.4
2	3.138	6.4	4.9	5.1	104.0	57.4	61.7
3	7.554	7.6	7.2	7.3	0.6	-4.5	-3.9
4	9.868	8.8	9.4	9.2	-10.8	-5.1	-6.5
5	14.137	10.0	11.4	11.0	-29.3	-19.5	-22.1
6	11.072	11.2	13.3	12.6	1.2	19.8	14.1
7	19.747	12.4	15.0	14.1	-37.2	-24.0	-28.6
8	8.556	13.6	16.6	15.4	59.0	94.5	80.2
9	19.422	14.8	18.1	16.6	-23.8	-6.6	-14.4
10	18.127	16.0	19.5	17.7	-11.7	7.6	-2.4
11	18.906	17.2	20.7	18.7	-9.0	9.7	-1.2
12	21.937	18.4	21.8	19.6	-16.1	-0.4	-10.8
13	24.017	19.6	22.8	20.4	-18.4	-5.0	-15.2
14	23.429	20.8	23.7	21.1	-11.2	1.0	-10.0
16	22.662	23.2	25.0	22.3	2.4	10.1	-1.4
17	21.465	24.4	25.4	22.9	13.7	18.4	6.6
18	25.440	25.6	25.7	23.4	0.6	1.2	-8.1
19	22.529	26.8	25.9	23.8	19.0	15.1	5.7
20	26.408	28.0	26.0	24.2	6.0	-1.5	-8.3
21	25.082	29.2	25.9	24.6	16.4	3.4	-2.0





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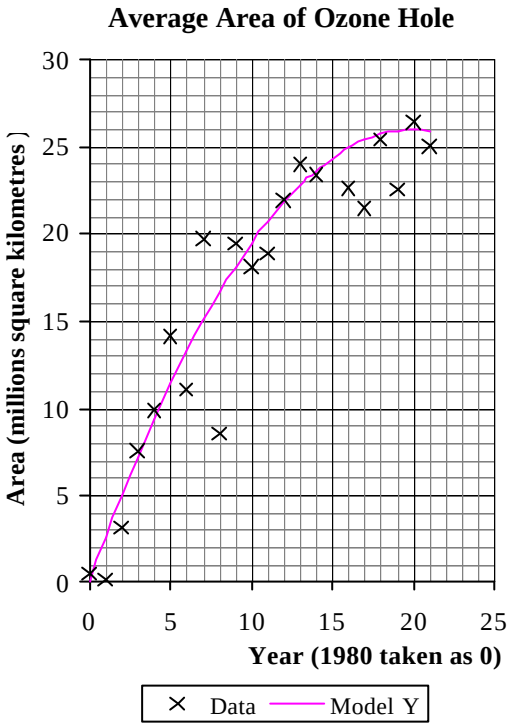

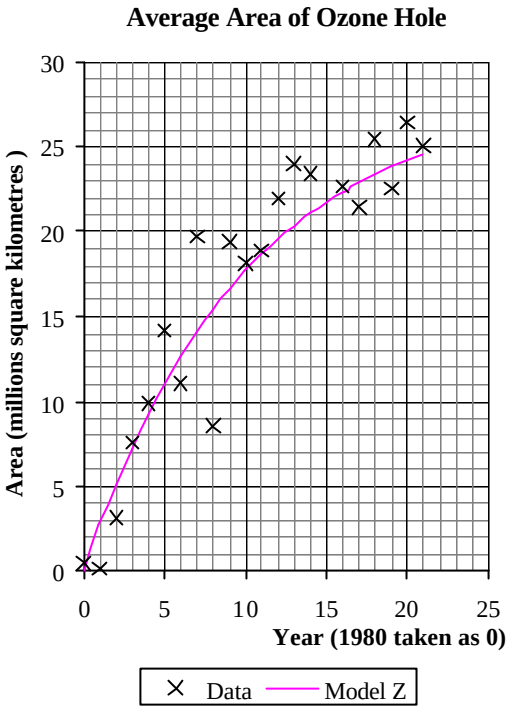

Predictions for Future Years

Minimum Ozone Level

Linear Model A: $y = 190 - 5.3t$ predicts that the minimum ozone level will steadily reduce to zero when $t \approx 36$ i.e. in about 2016

Quadratic Model B: $y = 0.28t^2 - 11t + 205$ predicts that the minimum ozone level was a minimum of approximately 97 DU in about 2000 after which it is rising and it will continue to rise in the future.

Exponential Model C: $y = 195e^{-0.04t}$ predicts that the minimum ozone level will reduce exponentially towards zero as time goes by.

Average Size of Ozone Hole

Linear Model X: $y = 1.2t + 4$ predicts that the average size of the ozone hole increases steadily, without limit as time goes by.

Quadratic Model Y: $y = 2.6t - 0.065t^2$ predicts that the average size of the ozone hole reached a maximum of 26 million square kilometres when $t = 20$ i.e. in 2000. The model predicts that the average size of the ozone hole is now reducing and will continue to reduce until it disappears when $t = 40$ i.e. in 2020

Exponential Model Z: $y = 28\left(1 - e^{-0.1t}\right)$ predicts that the average size of the ozone hole will approach a limit of 28 million square kilometres.

For further information about ozone visit the websites at:
http://toms.gsfc.nasa.gov/ozone/ozone.html (NASA)
http://www.atm.ch.cam.ac.uk (University of Cambridge Centre for Atmospheric Studies)
http://www.epa.gov/ozone/science/hole/sizedata.html (US Environmental Protection Agency)