Credit Cards

A credit card allows you to buy now and pay later, but there is usually a penalty for doing this – each month interest is added to what you owe. The rate of interest varies from one card to another, often being about 1 to 2%. This does not sound much, but it can accumulate to a substantial amount.

Suppose you buy a computer costing £1250 and pay using a credit card that charges interest at a rate of 1.2% per month. Assume that:

- you pay back £80 each month until the balance is less than £80 and then you make one final payment to clear the debt
- you buy nothing more with the credit card.

The amount owed at the end of each month
= Amount owed at the beginning of the month + Interest – Amount paid

Suppose £B_n denotes the balance (i.e. the amount owed) at the end of the n^{th} month.
This is also the amount owed at the start of the $(n + 1)^{th}$ month.

The amount owed at the end of the $(n + 1)^{th}$ month is $B_{n+1} = B_n + 0.012B_n - 80$

This can be simplified to: $B_{n+1} = 1.012B_n - 80$ …….(1)

Relationships like this are called **recurrence relations.** Given a starting value, B_0, you can use the recurrence relation over and over again to find the subsequent values B_1, B_2, B_3, ….

In this case the 'starting value', B_0 is the initial debt of £1250.

Using this as B_0 gives
$$B_1 = 1.012 \times 1250 - 80 = 1185$$
$$B_2 = 1.012 \times 1185 - 80 = 1119.22$$
$$B_3 = 1.012 \times 1119.22 - 80 = 1052.65$$ and so on.

It may seem a tedious job to carry out a long sequence of such calculations to find out when the debt will be paid off, but it can be done quickly using the Answer key on a calculator.

Spreadsheets also provide a quick method for recurrence relations if you use the 'Fill down' command to find successive values.

Try one, or both, of these methods as explained on the following pages.

Using a graphic calculator

- Enter the value of B_0, in this case 1250, into your calculator.
- Now enter the recurrence relation $B_{n+1} = 1.012B_n - 80$ using the Ans key as B_n i.e. $1.012Ans - 80$
- Repeatedly press the 'equals' key to give successive terms in the sequence. Check that the values you get agree with those given below and complete the table.

End of Month	Balance
0	£1,250.00
1	£1,185.00
2	£1,119.22
3	£1,052.65
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	

At the end of the 17th month you should find that there is only £32.28 left to pay off.

If you pay this off then, how much will your computer have cost altogether?

How much will you have paid in interest?

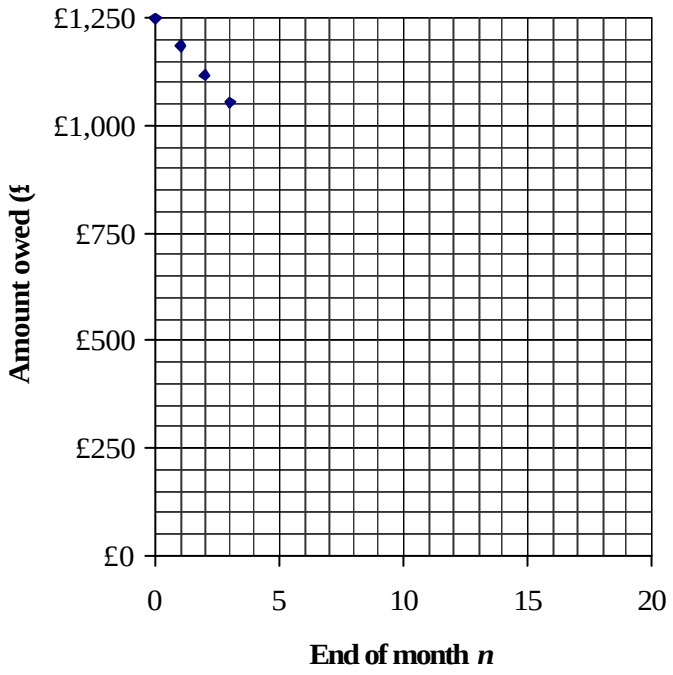
Express the interest as a percentage of the original price.

The way in which the debt decreases can be shown on a graph.

Complete this graph.

Do the points lie on a straight line or a curve?

Amount owed on credit card



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Using a spreadsheet

The recurrence relation that gives the amount owed at the end of each month is $B_{n+1} = 1.012B_n - 80$ The way this is entered into a spreadsheet is shown below. Compare the formulae used in cells B3, B4 and B5 with the recurrence relation.

Copy the contents of cells A1 to B3 onto a spreadsheet and use 'Fill down' to extend the table.

	A	B
1	**End of Month**	**Balance**
2	0	1250
3	=A2+1	=1.012*B2-80
4	=A3+1	=1.012*B3-80
5	=A4+1	=1.012*B4-80

You should find that the debt reduces to £32.28 at the end of the 17th month.

If you pay this then, how much will your computer have cost altogether?

How much will you have paid in interest?

Express the interest as a percentage of the original price.

Use columns A and B to plot a graph of the amount owed against time.
Do the points lie on a straight line or a curve?

Comparing credit cards and paying off debts

Search the internet for information about credit cards. You may find that some credit cards have special offers such as a 0% interest charge for the first six months.

Imagine you are buying something expensive using a credit card. Use recurrence relations to investigate how long it would take to pay off the debt and how this varies with different credit cards and different regular payments.

Write a summary of what you find.

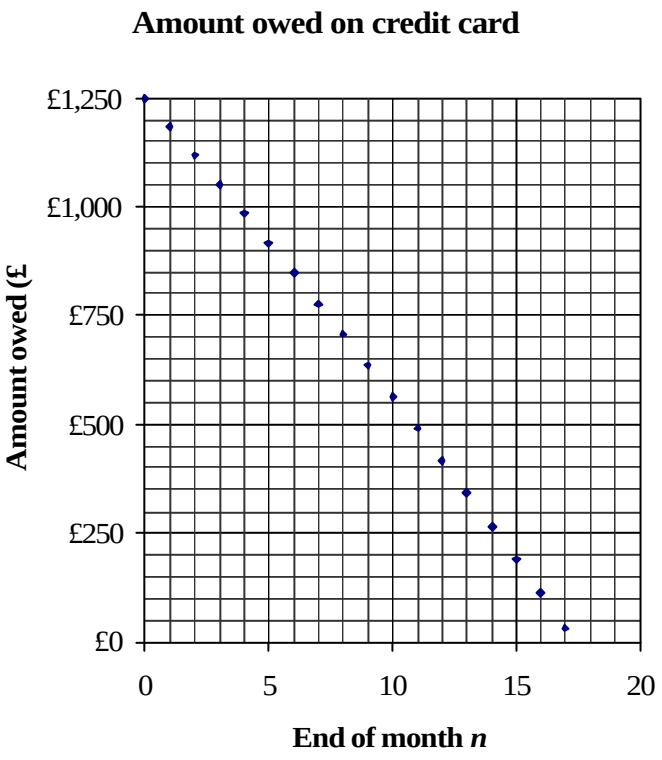
Teacher Notes

Units
Advanced Level, Mathematical Principles of Personal Finance & Applying Mathematics

Notes
The worksheets on pages 1 to 3 show how a recurrence relation can be used on a graphic calculator and a spreadsheet to find out how long it would take to pay off a credit card debt. You may decide to use one or both of these methods. The accompanying Excel spreadsheet can be used to demonstrate the spreadsheet methods or formulae.
The last part of the worksheet asks learners to investigate how long it would take to pay off a debt and how this varies with different credit cards and different regular payments. Before they do this you may wish to discuss with them complications that could arise eg what to do when credit cards have an initial 0% interest period. You may wish to provide the information for students or ask them to find information for themselves from the internet. As well as the major banks' and building societies' websites, there are many other websites that give information about credit cards. For example, the independent website www.moneyfacts.co.uk provides a wide range of information and links to other relevant websites.

Answers The completed table and graph are given below.

End of Month	Balance
0	£1,250.00
1	£1,185.00
2	£1,119.22
3	£1,052.65
4	£985.28
5	£917.11
6	£848.11
7	£778.29
8	£707.63
9	£636.12
10	£563.75
11	£490.52
12	£416.40
13	£341.40
14	£265.50
15	£188.68
16	£110.95
17	£32.28



Total cost = $17 \times £80 + £32.28 = £1392.28$
Interest paid = £1392.28 – £1250 = £142.28.
As a % of the original price, this is $\frac{142.28}{1250} \times 100 = 11.4\%$