Coughs and Sneezes

Assignment

At the beginning of a term it is noticed that a large number of the university students who live in a particular hall of residence have a cold. The way in which the cold spreads is monitored by recording the number of students suffering from colds every five days.
The results are given in the table below where t represents the number of days after monitoring began and s represents the number of students who have a cold.

t	0	5	10	15	20	25	30	35	40	45	50	55	60	65	70
s	25	31	38	43	47	47	45	41	36	30	24	19	14	11	8

Find:
- a trigonometrical function that can be used to model the full data set;
- a polynomial function that can be used to model the data for values of t between 0 and 50.

Compare your models with each other and with the original data.

In your report you should:
- choose appropriate models
- explain how you chose the parameters of your functions referring to how they relate to basic functions of their type;
- show clearly the stages of your working when using algebraic or trigonometric techniques;
- use a graphic calculator or computer software to compare graphs of your functions with a graph of the original data;
- consider the effectiveness of each function as a model;
- use your graphs of functions to predict what will happen in cases for which you have no data;
- consider how inaccuracies in the data may affect your models by considering how, in general terms, the functions you found could be different.

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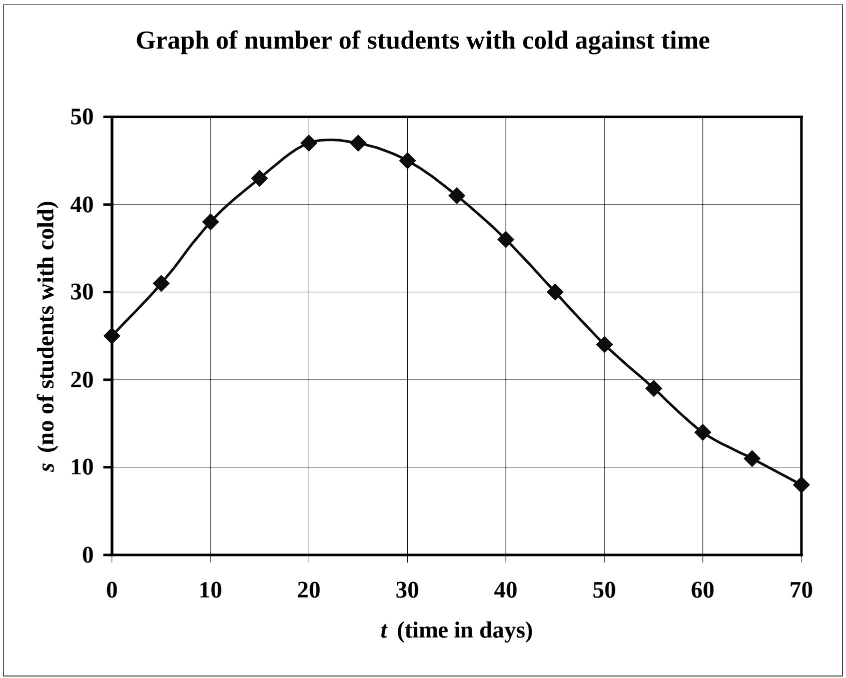
Teacher Notes

Unit Advanced Level, Working with algebraic and graphical techniques

Notes
An Excel spreadsheet has been used to give the graph below showing the given data**.**



Students are expected to use a sine function as the trigonometric model and a quadratic function as the polynomial model. However there are other possibilities and some students may choose to use a cosine function and/or a higher degree polynomial.

There are a variety of ways in which the parameters can be evaluated and there will be a variety of acceptable models.

For example, in the case of a quadratic function one method is to substitute data pairs into the form $s = at^2 + bt + c$ then use simultaneous equations.
Using the points (0, 25), (25, 47) and (50, 24) in this way gives the quadratic function $s = 25 + 1.78t - 0.036t^2$

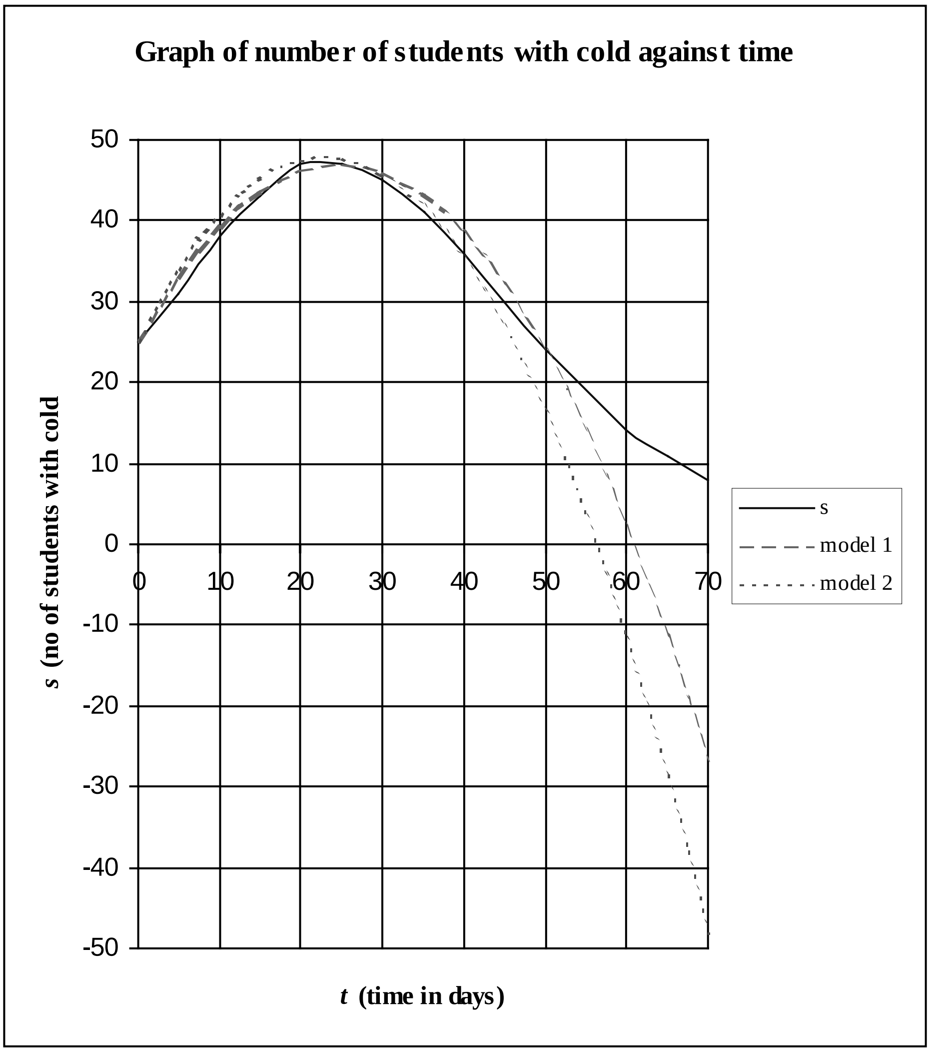
An alternative method is to use transformations of the curve $s = t^2$ to build up a model in the form $s = a(t+b)^2 + c$.
Suppose the maximum point is taken to be (23, 48). Then starting with $s = at^2$ a reflection in the *t* axis, translation of 23 in the *t* direction and 48 in the *s* direction would give the function $s = 48 - a(t-23)^2$. Substituting (0, 25) then gives $a = 0.0435$ and the function becomes $s = 48 - 0.0435(t-23)^2$

So we have two possible models:

Model 1 $s = 25 + 1.78t - 0.036t^2$

Model 2 $s = 48 - 0.0435(t - 23)^2$

The graph below shows the original data and the two models for $0 \le t \le 70$



The graph shows that for some values of *t* Model 1 is the better model and for other values of *t* Model 2 is the better model. Neither function gives a good model for large values of *t*.

Similarly there are a variety of possible methods for finding a trigonometric model. Using the form $s = a \sin \omega t + c$ and taking the maximum point at (23, 48) gives the function $s = 23 \sin 0.0683t + 25$. Other assumptions will lead to different trigonometric forms.

The sketch graph on the following page shows the following models:

Model 3 $s = 23 \sin 0.0683t + 25$

Model 4 $s = 23 \sin 0.0633t + 25$

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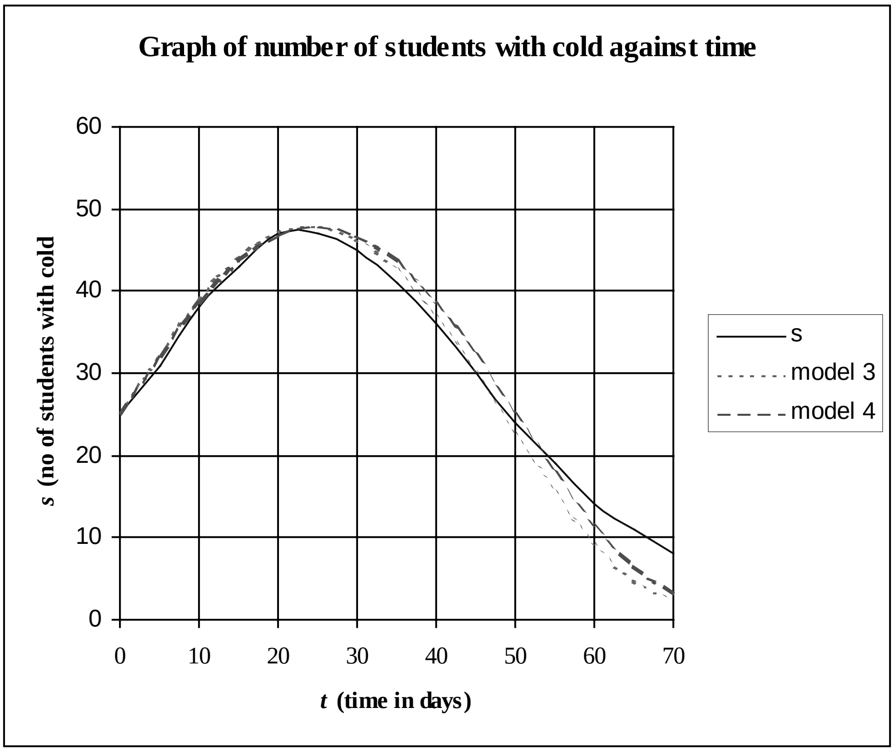
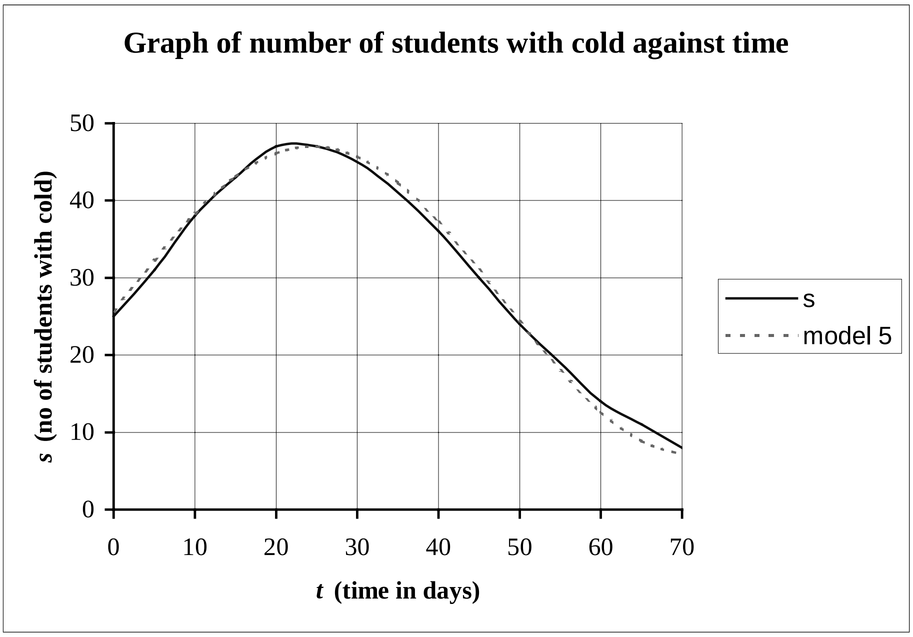
Both functions provide good models for $0 \le t \le 25$ but are less good for $25 \le t \le 70$, sometimes Model 3 being the better model and sometimes Model 4.



If the more general trigonometric form $s = a\sin(\omega t + \varepsilon) + c$ is used, models can be found which follow the real data more closely for $25 \le t \le 70$.
Model 5 shown below is the function $s = 20\sin(0.0668t - 0.0668) + 27$



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