Two at a Time

The digits in our number system are: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
Follow the instructions given below and see what happens.

Two digits

- Take any two digits.
- Find the sum of the digits.
- Make as many two-digit numbers as you can, without repeating digits.
- Find the sum of the two-digit numbers.
- Look for a relationship between the sum of the two-digit numbers and the sum of the digits.

Example

5 and 9
5 + 9 = **14**
  59
  95
154
Can you see any relationship between **14** and **154**?

Whether you can see a relationship or not, follow the steps again with different pairs of digits.

Can you make a general statement about what you have found?

Now use a starting group of three digits:

Three digits

- Take any three digits.
- Find the sum of the digits.
- Without repeating digits, make as many two-digit numbers as you can from the three digits in the group.
- Find the sum of the two-digit numbers.
- Look for a relationship between the sum of the two-digit numbers and the sum of the digits.

Example

2 and 6 and 8
2 + 6 + 8 = **16**
  26
  62
  28
  82
  68
  86
352
Can you see any relationship between **16** and **352**?

Try this again starting with different groups of three digits.

Can you make a general statement about what you have found?

Extend the investigation.
Try to explain or prove the relationships you find.

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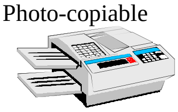

Teacher Notes

Unit Intermediate Level, Making connections in mathematics

Notes
This assignment can be used to give evidence for portfolio requirement 3, given below:

3 A report of an enquiry based in the area of **either** number **or** geometry that includes (i) illustration of your findings referring to particular cases and your conclusions that apply to the general case; (ii) your reasoning as to why your conclusions are correct.	

Answers
Two digits
The relationship is $T = 11S$ where T is the sum of the two-digit numbers and S the sum of the digits.
This can be proved using algebra:
Suppose the two digits are a and b.
Sum of digits $S = a + b$
Values of possible two-digit numbers are $10a + b$ and $10b + a$
Sum of these two-digit numbers $T = 11a + 11b = 11(a + b) = 11S$

Three digits
In this case $T = 22S$. To prove this:
Suppose the three digits are a, b and c
Sum of digits $S = a + b + c$
Possible two-digit numbers are: $10a + b$, $10b + a$, $10a + c$, $10c + a$, $10b + c$, $10c + b$
Sum of these two-digit numbers $T = 22a + 22b + 22c = 22(a + b + c) = 22S$

Increasing the size of the group of digits used
The obvious way to extend the investigation is to increase the size of the group of digits used.
For a starting group of four digits the relationship is $T = 33S$
In general, for a starting group of n digits the relationship is $T = 11(n-1)S$ for any value of n from 2 to 10.

Extending the length of the numbers
The investigation can also be extended by increasing the length of the numbers from two-digit numbers to three-digit numbers then four-digit numbers (etc). This becomes increasingly difficult. The results are given below, but it is not expected that students will find many of these.

If as many three-digit numbers as possible are made from a starting group of three digits, the relationship between the sum of the digits, S, and the sum of all the possible three-digit numbers, T, is $T = 222S$
If three-digit numbers are made from a starting group of four digits, the relationship is $T = 666S$

In general, using three-digit numbers from a starting group of n digits, $T = (n-1)(n-2) \times 111S$
(for $3 \le n \le 10$).

The general result for r-digit numbers from a starting group of n digits is $T = \frac{{}^nP_r}{n} \times 11...1S$
(for $2 \le r \le n \le 10$) where the term 11…1 has r digits, each of these digits being 1

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