

# Formulae

Fixed values such as the number of days in a week (i.e. 7) are called **constants**.

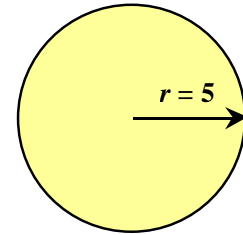
Values that are not fixed are called **variables**. Letters can be used to represent variables and relationships between variables can be summarised using **formulae**.

**Substituting** particular values into a formula gives information about particular cases.

## Example

The area of a circle of radius  $r$  is given by the formula  $A = \pi r^2$  ( $\pi$  is a constant whose value is approximately 3.14 but a more accurate value is programmed into scientific calculators).

Find the area of a circle with radius 5 cm.



## How to do it....

Substituting  $r = 5$  into the formula for the area of a circle gives:

$$A = \pi \times 5^2 = \pi \times 5 \times 5 = 78.539 \dots$$

So the area of the circle whose radius is 5 cm is **79 cm<sup>2</sup> (to the nearest cm<sup>2</sup>)**.

## Example

When an amount of money  $\pounds P$  is left in a building society account that gives  $r\%$  interest per annum,

the amount in the account after  $n$  years is given by the formula  $A = P \left( 1 + \frac{r}{100} \right)^n$ .

If  $\pounds 750$  is invested in an account giving 4.5% per annum interest, find the amount in the account after 6 years.

## How to do it....

Substitute  $P = 750$ ,  $r = 4.5$  and  $n = 6$  into  $A = P \left( 1 + \frac{r}{100} \right)^n$  giving:

$$A = 750 \left( 1 + \frac{4.5}{100} \right)^6 = 750 \times 1.045^6 = 976.695 \dots$$

The amount in the account after 6 years is **£976.70 (to the nearest pence)**.

## Example

The radius of a sphere is  $r = \sqrt[3]{\frac{3V}{4\pi}}$  where  $V$  is its volume.

Find the radius of a ball bearing whose volume is  $9.6 \text{ mm}^3$

## How to do it....

Substitute  $V = 9.6$  into  $r = \sqrt[3]{\frac{3V}{4\pi}}$ , giving  $r = \sqrt[3]{\frac{3 \times 9.6}{4 \times \pi}} = \sqrt[3]{2.29183 \dots} = 1.318 \dots$

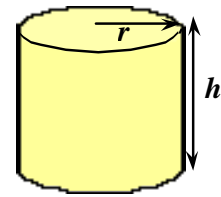
The radius of the ball bearing is **1.3 mm (to 2 sf)**



**Try these.....**

- 1 The volume of a cylinder is given by the formula  $V = \pi r^2 h$  where  $r$  is the radius and  $h$  is the height.

Find the volume of a cylindrical can that has radius 4 cm and height 7.5 cm.



- 2 The speed of a car after  $t$  seconds is given by the formula  $v = u + at$  where  $u$  is the starting speed in metres per second ( $\text{m s}^{-1}$ ) and  $a$  is the acceleration in metres per second per second ( $\text{m s}^{-2}$ ).

A car joins a motorway at a speed of  $25 \text{ m s}^{-1}$  and accelerates at  $0.65 \text{ m s}^{-2}$  for 12 seconds.

Find its speed at the end of this time.

- 3 If a ball is thrown up in the air at a velocity of  $v \text{ m s}^{-1}$ , the height it reaches is given by  $h = \frac{v^2}{2g}$

where  $g$  is the acceleration due to gravity in  $\text{m s}^{-2}$ . (NB  $g$  is approximately 9.81)

Find the height reached by a ball thrown up at a velocity of  $15 \text{ m s}^{-1}$ .

- 4 The formula for converting temperatures is  $C = \frac{5(F - 32)}{9}$

where  $F$  is the temperature in degrees Fahrenheit and  $C$  is the temperature in degrees Celsius.

An old recipe gives the oven temperature  $375^\circ \text{ F}$ . Convert this temperature to Celsius.

- 5 When an object is released so that it falls to the ground it reaches a velocity given by  $v = \sqrt{2gh}$  where  $h$  is the height (in metres) from which it is dropped and  $g$  is approximately 9.81

If a stone is dropped from bridge at a height of 50 metres above a river, find its velocity on impact.

- 6 When a car accelerates from a speed of  $u \text{ m s}^{-1}$  to a speed of  $v \text{ m s}^{-1}$  in  $t$  seconds, the distance it travels (in metres) is given by the formula  $d = \frac{t(u + v)}{2}$ .

Find the distance travelled by a car that accelerates from  $17.5 \text{ m s}^{-1}$  to  $32 \text{ m s}^{-1}$  in 9.5 seconds.

- 7 The time taken for a pendulum to make one full swing is  $T = 2\pi \sqrt{\frac{l}{g}}$

where  $l$  metres is the length of the pendulum and  $g$  is approximately 9.81

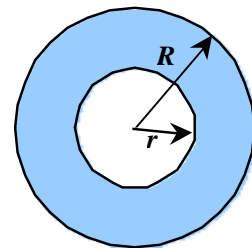
Find the time it takes a pendulum of length 0.5 metres to make a full swing.

- 8 If the monthly rate of interest on a loan is  $m\%$ , the annual percentage rate (APR) is given by:

$$\text{APR} = 100 \left[ \left( 1 + \frac{m}{100} \right)^{12} - 1 \right]. \text{ Find the APR if the monthly rate is } 1.25\%.$$

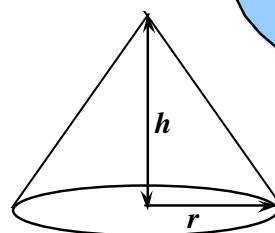
- 9 The area of a washer is given by the formula  $A = \pi(R^2 - r^2)$  where  $r$  is the inner radius and  $R$  is the outer radius.

Find the area when  $r = 5 \text{ mm}$  and  $R = 7.5 \text{ mm}$ .



- 10 The formula  $A = \pi r \sqrt{r^2 + h^2}$  gives the area of the curved surface of a cone with radius  $r$  and height  $h$ .

Find  $A$  when  $r = 7.5 \text{ cm}$  and  $h = 8.2 \text{ cm}$ .



**Teacher Notes**

**Unit** Intermediate Level, Using algebra, functions and graphs

**Skills used in this activity:**

- substituting values into formulae
- using a calculator to evaluate expressions.

**Preparation**

Students need to be able to use a calculator to evaluate expressions involving brackets, powers and roots. The accompanying Powerpoint presentation could be used for class discussion about formulae and to go through the examples of this type before students try some themselves. This presentation can be adapted to include more or fewer examples.

**Answers (to 3 sf)**

- 1  $377 \text{ cm}^3$
- 2  $32.8 \text{ m s}^{-1}$
- 3  $11.5 \text{ m}$
- 4  $191^\circ \text{ C}$
- 5  $31.3 \text{ m s}^{-1}$
- 6  $235 \text{ m}$
- 7  $1.42 \text{ seconds}$
- 8  $16.1\%$
- 9  $98.2 \text{ cm}^2$
- 10  $262 \text{ cm}^2$

