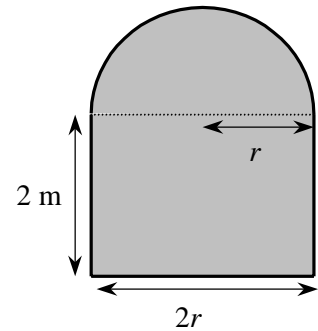


Road Tunnel



A road tunnel is designed to have a cross section that consists of a rectangle surmounted by a semi-circle as shown in the sketch.

The height of the side of the tunnel is to be 2 metres.

The area of the cross section must be at least 16 m^2 to allow adequate ventilation. The strength of the materials used to support the tunnel suggests that the area must be no more than 32 m^2 because of the danger of collapse.

The designer needs to know how these constraints affect the width of the tunnel.

In this activity you will use a graph to find the minimum and maximum values of the radius, r metres, and then the range of possible road widths. First we need a formula for the area in terms of r .

The area of the semi-circle is $0.5\pi r^2$ The area of the rectangle is $2 \times 2r = 4r$

So the formula for the area of the tunnel entrance is $A = 0.5\pi r^2 + 4r$

A graph can be drawn in Excel to show the area of the tunnel entrance for different values of r . Follow these instructions to draw the graph of A against r for values of r up to 4 metres.

How to do it...

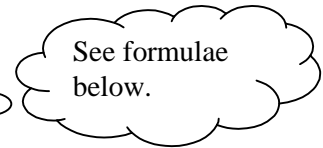
- 1 Use the formula to *set up a spreadsheet table*.

Put headings r (m) and A (m^2) at the top of columns A and B.

Use 0 as the first value of r . Enter 0 in cell A2.

Write a spreadsheet formula in cell B2 to calculate the area.

Write a spreadsheet formula in cell A3 to calculate the next value of r .



	A	B	C
1	r (m)	A (m^2)	
2	0	0	
3	0.5		

Numerical

	A	B	C
1	r (m)	A (m^2)	
2	0	$=0.5*PI()*A2^2+4*A2$	
3	$=A2+0.5$		

Formulae

- 2 Use *'fill down'* to complete the table as far as $r = 4$. The results are shown below with the r column formatted to show 1 decimal place and the A column to show 2 decimal places:

	A	B	C
1	r (m)	A (m^2)	
2	0.0	0.00	
3	0.5	2.39	
4	1.0	5.57	
5	1.5	9.53	
6	2.0	14.28	
7	2.5	19.28	
8	3.0	26.14	
9	3.5	33.24	
10	4.0	41.13	

Numerical

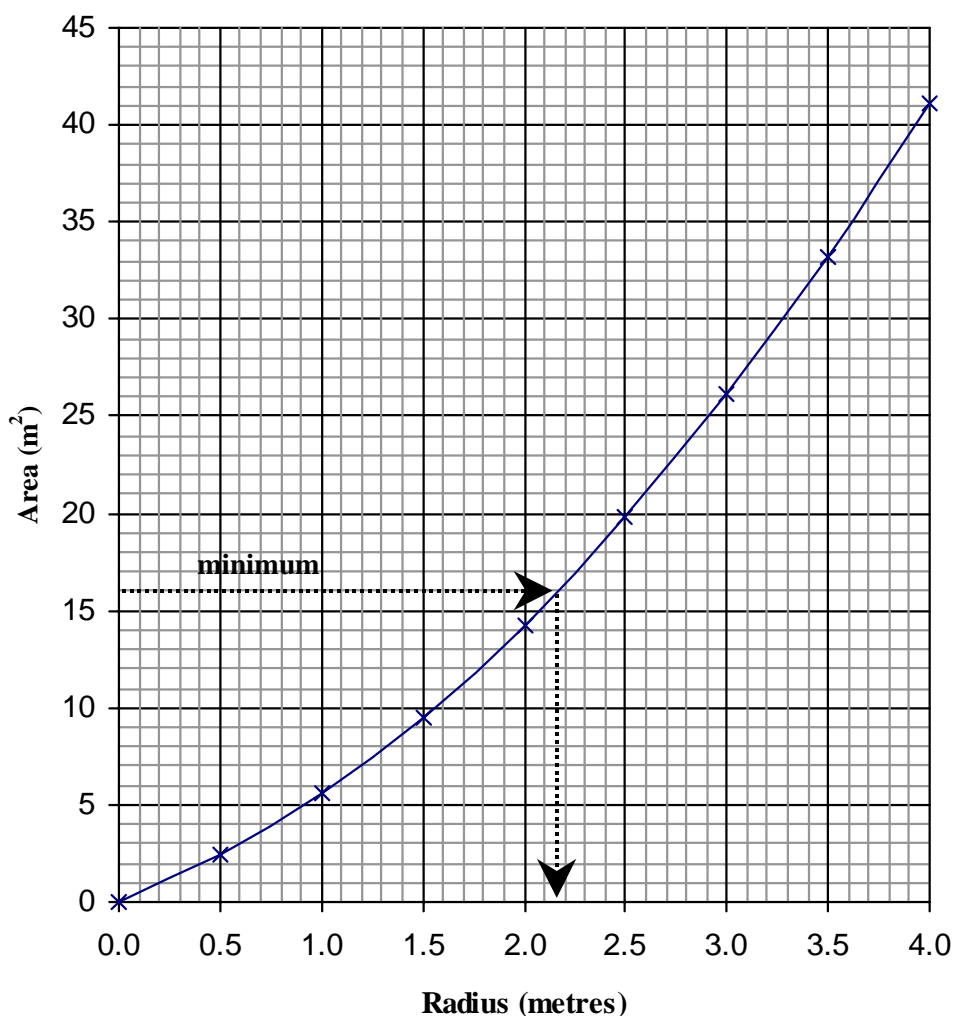
	A	B	C
1	r (m)	A (m^2)	
2	0.0	$=0.5*PI()*A2^2+4*A2$	
3	$=A2+0.5$	$=0.5*PI()*A3^2+4*A3$	
4	$=A3+0.5$	$=0.5*PI()*A4^2+4*A4$	
5	$=A4+0.5$	$=0.5*PI()*A5^2+4*A2$	
6	$=A5+0.5$	$=0.5*PI()*A6^2+4*A2$	
7	$=A6+0.5$	$=0.5*PI()*A7^2+4*A2$	
8	$=A7+0.5$	$=0.5*PI()*A8^2+4*A2$	
9	$=A8+0.5$	$=0.5*PI()*A9^2+4*A2$	
10	$=A9+0.5$	$=0.5*PI()*A10^2+4*A2$	

Formulae



- 3 To draw the graph, highlight the values in columns A and B and use the Chart Wizard to **draw a scatter graph** (choose the curves with points). When setting up the graph, write in a title and labels, remove the legend and choose to use both major and minor gridlines.
- 4 **Format the gridlines** so that it is easy to read off values of A and r .
- 5 The graph can be used to find the minimum and maximum values of r :
 - a The area of the tunnel entrance should be at least 16 m^2 to allow adequate ventilation. The minimum value of r is about 2.16 metres as shown below. Check this on your graph. The road width is $2r$. What is the minimum road width?

Area of Tunnel Cross Section

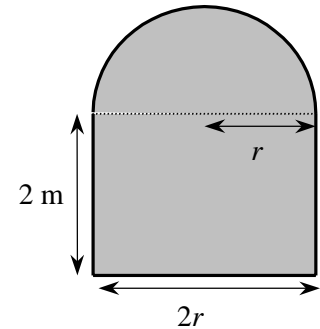


- b The maximum area is 32 m^2 . Use the graph to find the maximum value of r . What is the maximum road width?

Draw lines on your graph to show the maximum and minimum values of r . You can print your graph then do this by hand, or use the drawing tools in Excel before printing the graph.



Road Tunnel



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The height of the side of the tunnel is to be 2 metres.

The area of the cross section must be at least 16 m^2 to allow adequate ventilation, but the strength of the materials used to support the tunnel suggests that the area must be no more than 32 m^2 because of the danger of collapse.

The designer needs to know how these constraints affect the width of the tunnel.

The problem can be solved using quadratic equations.

First the radius that will give an area of 16 m^2 can be found as shown below:

How to do it...

The area of the semi-circle is $0.5\pi r^2$ and the area of the rectangle is $2 \times 2r = 4r$

so the formula for the area of the tunnel entrance is $A = 0.5\pi r^2 + 4r$

For an area of 16 m^2 $0.5\pi r^2 + 4r = 16$

This rearranges to $0.5\pi r^2 + 4r - 16 = 0$

In the quadratic formula: $a = 0.5\pi$, $b = 4$, $c = -16$

Using the formula:
$$r = \frac{-4 \pm \sqrt{4^2 - 4 \times 0.5 \times \pi \times -16}}{2 \times 0.5 \times \pi}$$

$$r = \frac{-4 \pm \sqrt{116.53}}{\pi} = \frac{-4 \pm 10.795}{\pi}$$

The radius must be positive so $r = \frac{-4 + 10.795}{\pi} = \frac{6.795}{\pi} = 2.163$

This gives a minimum value for the radius of 2.163 metres.

The minimum tunnel width = $2r = 2 \times 2.163 = 4.33$

The minimum width of the road tunnel is 4.3 metres (to 1 decimal place).

Use a similar method to find the value of r that will give an area of 32 m^2

Use this value of r to find the maximum width of the tunnel.

Quadratic Formula

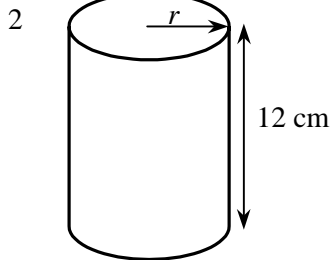
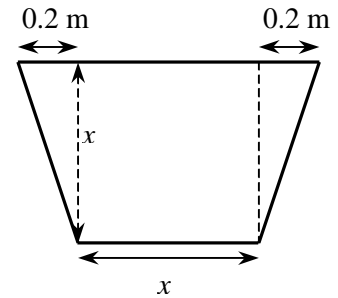
Solutions of $ax^2 + bx + c = 0$

are: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$



Now solve these:

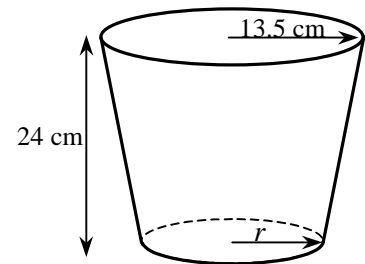
- 1 The sketch shows the cross section of a design for a waste skip.
- a Show that the area of the cross section is given by the formula:
 $A = x^2 + 0.2x$ where x is the length of the base and height in metres.
- b In order that the skip should have the required volume, the cross sectional area must be 2.5 square metres. Find the value of x .



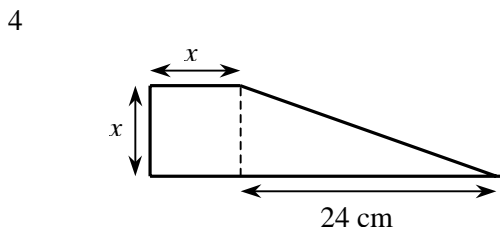
A container is to be in the shape of a cylinder of height 12 cm.

- a Explain why the total surface area of the container is given by:
 $A = 2\pi r^2 + 24\pi r$
- b The manufacturers want to limit the surface area of the container to 300 cm^2 . Find the maximum radius.

- 3 The formula for the volume of a bucket of height h is
 $V = \frac{1}{3}\pi h(R^2 + Rr + r^2)$ where R and r are the radii of the ends.



- a A bucket is designed to be 24 cm high and to have a top with radius 13.5 cm. Show that for this bucket:
 $V = 8\pi(182.25 + 13.5r + r^2)$
- b The bucket is required to have a volume of 10 litres where 1 litre = 1000 cm^3 . Find the radius of the bottom of the bucket.

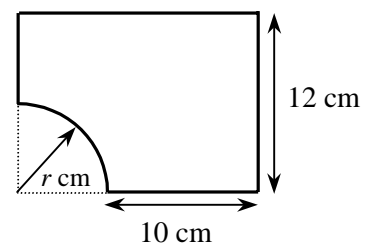


The sketch shows the cross section of a wedge.

Find the value of x that would give a cross-sectional area of:

- a 150 cm^2 b 275 cm^2

- 5 A quarter circle is to be removed from a rectangular metal plate to give the shape shown in the sketch.



- a Show that the remaining area is $A = 120 + 12r - 0.25\pi r^2$
- b It is required that the area should be 140 cm^2 . Find the value of r .



Teacher Notes

Unit Intermediate Level, Using algebra, functions and graphs

Notes

This resource gives two methods for solving a problem involving a quadratic function. You could use both methods together, or split the materials and use the two methods at different times in the course.

The first method involves using Excel to draw a quadratic graph. Students will need to be familiar with the use of functions and fill down in Excel and also know how to format graphs.

The second method involves the use of the quadratic formula. This part of the resource could be used as a follow-up to another Nuffield resource called 'The Quadratic Formula'.

The sheet of mixed problems can be solved using either method.

Answers**Road Tunnel****Using the Graph:**

Minimum road width is 4.3 m (to 1 dp).

Maximum value of $r \approx 3.42$ metres, giving a maximum road width of 6.8 metres (to 1 dp).

Using the Quadratic Formula:

For an area of 32 m^2 $0.5\pi r^2 + 4r = 32$ which rearranges to $0.5\pi r^2 + 4r - 32 = 0$

In the quadratic formula: $a = 0.5\pi$, $b = 4$, $c = -32$

Using the formula:
$$r = \frac{-4 \pm \sqrt{4^2 - 4 \times 0.5 \times \pi \times -32}}{2 \times 0.5 \times \pi}$$

$$r = \frac{-4 \pm \sqrt{217.06}}{\pi} = \frac{-4 \pm 14.733}{\pi}$$

The radius must be positive so $r = \frac{-4 + 14.733}{\pi} = \frac{10.733}{\pi} = 3.416$

The tunnel width = $2r = 2 \times 3.416 = 6.83$

The maximum width is 6.8 metres (to 1 decimal place).

Other Problems.

- 1 b 1.5 m (to 1 dp)
- 2 b 3.2 cm (to 1 dp)
- 3 b 9.4 cm (to 1 dp)
- 4 a 7.6 cm (to 1 dp) b 11.6 cm (to 1 dp)
- 5 1.9 cm (to 1 dp)

