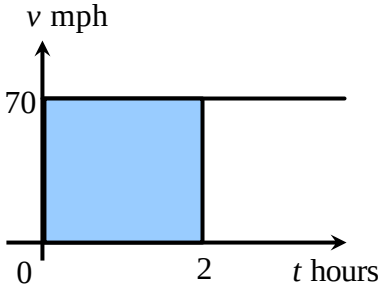
Area under a graph

The area under a graph often gives useful information.

Velocity-time graphs

Constant velocity

The sketch shows the velocity-time graph for a car that is travelling along a motorway at a steady 70 mph.



The area under this graph is rectangular in shape.

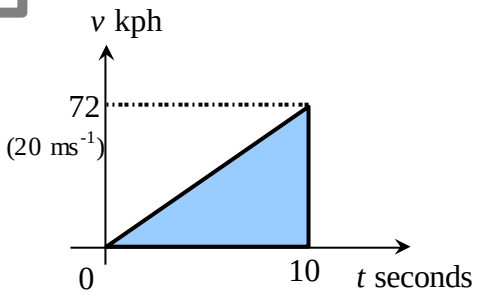
The shaded area = $2 \times 70 = 140$

This is the distance in miles travelled in 2 hours when the speed is 70 mph.

Area under a velocity-time graph = distance travelled.

Constant acceleration

This graph shows the velocity of a car accelerating steadily from 0 to 72 kilometres per hour (kph) in 10 seconds.



Note that the velocity is in kilometres per ***hour*** whilst the time is in ***seconds***. The units need to be converted in order to find the distance travelled.

$$72 \text{ kph} = \frac{72 \times 1000}{60 \times 60} = 20 \text{ metres per second (ms}^{-1}\text{)}.$$

(1 km = 1000 m)

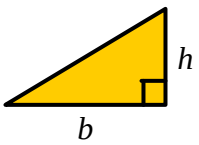
The distance travelled is given by the area under the graph:

$$\text{Distance travelled} = \frac{10 \times 20}{2} = 100 \text{ metres.}$$

Note this is equivalent to the car travelling for 10 seconds at the average speed of 10 metres per second.

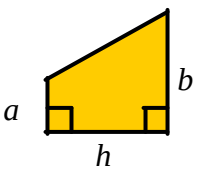
Formulae

Area of a triangle $= \dfrac{\text{base} \times \text{height}}{2} = \dfrac{bh}{2}$

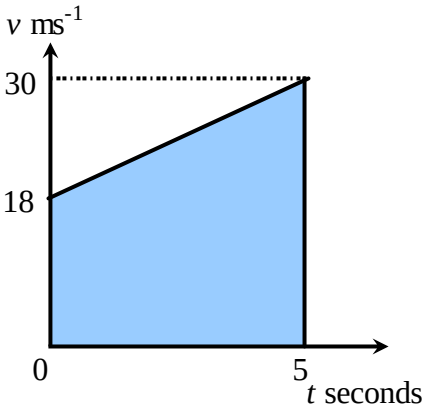


Area of a trapezium $= \dfrac{\text{sum of parallel sides} \times \text{distance between}}{2}$

$$= \frac{(a+b)h}{2}$$



This graph represents a car accelerating steadily from 18 to 30 ms^{-1} in 5 seconds.



The shaded area is a trapezium with area = $\frac{(18+30)\times 5}{2}$

The distance travelled = 120 metres

(Note this is equivalent to the car travelling for 5 seconds at an average speed of 24 ms^{-1})

In each case sketch a velocity-time graph and find the distance travelled:

1 Car travels at a steady velocity of 65 mph for 3 hours.

2 Car decelerates steadily from a velocity of 54 kph until it stops 4 seconds later.

3 Car accelerates steadily from 12 ms^{-1} to 24 ms^{-1} over a period of 6 seconds.

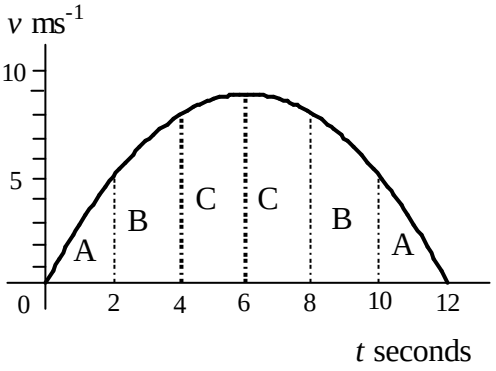
4 Car decelerates steadily from 108 kph to 63 kph over a period of 8 seconds.

Variable velocity and acceleration

The table and graph give the velocity of a car as it travels between 2 sets of traffic lights.

***t* (s)**	0	2	4	6	8	10	12
***v* (ms^{-1})**	0	5	8	9	8	5	0

An ***estimate*** for the area under this graph can be found by splitting it into strips as shown.



The strips at each end are approximately triangular in shape and each strip between them is approximately in the shape of a trapezium.

$$\text{Area of A} \approx \frac{2\times 5}{2} = 5$$

$$\text{Area of B} \approx \frac{(5+8)2}{2} = 13$$

$$\text{Area of C} \approx \frac{(8+9)2}{2} = 17$$

Total area $\approx 5 + 13 + 17 + 17 + 13 + 5$
$= 70$

The distance travelled by the car between the 2 sets of traffic lights is approximately 70 metres.

This is a simplified model of this situation. In practice the change in velocity is unlikely to be so smooth and symmetrical.

Note that better estimates can be found by using more data and narrower strips.

Velocity can be modelled by a function

Example

The velocity of a car, v ms^{-1} is modelled by the function $v = 0.5t^3 - 3t^2 + 16$ for $0 = t = 4$.

(a) Draw a velocity-time graph.

(b) Describe what happens.

(c) Estimate the distance travelled.

(a)

t (s)	0	1	2	3	4
v (ms^{-1})	16	13.5	8	2.5	0

(c) Area under graph $\approx \frac{(16+13.5)1}{2} + \frac{(13.5+8)1}{2} + \frac{(8+2.5)1}{2} + \frac{1\times 2.5}{2}$

$= 14.75 + 10.75 + 5.25 + 1.25$

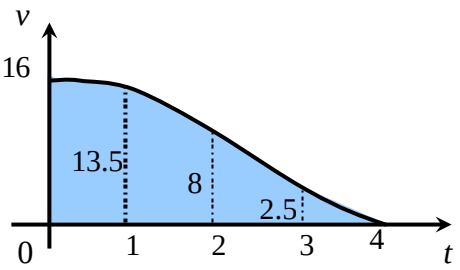
$= 32$

Distance travelled $\approx$ 32 metres

Note that using more strips would give a more accurate answer.

Note the graph can be drawn on graph paper by hand or on a graphic calculator or a spreadsheet.

$v = 0.5t^3 - 3t^2 + 16$



(b) The graph shows that the car slows down from 16 ms^{-1} and stops after 4 seconds.

Try these: In each of the following:

- draw a velocity-time graph (on graph paper, a graphic calculator or a spreadsheet)
- describe what happens during the given time interval
- estimate the distance travelled to the nearest metre.

1

t (s)	0	1	2	3	4	5	6	7	8
v (ms^{-1})	0	5.0	7.1	8.7	10.0	11.2	12.2	13.2	14.1

2

t (s)	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
v (ms^{-1})	5.0	6.9	8.6	10.1	11.4	12.5	13.4	14.1	14.6	14.9	15.0

3

t (s)	0	2	4	6	8	10
v (ms^{-1})	25	16	9	4	1	0

4

t (s)	0	1	2	3	4	5	6	7	8	9
v (ms^{-1})	20.0	14.0	11.5	9.6	8.0	6.6	5.3	4.1	3.0	2.0

5 Velocity of a car, v ms^{-1} modelled by $v = 0.15t^2$ for $0 = t = 8$.

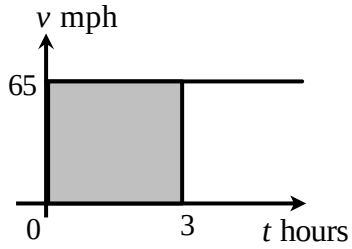
6 Velocity of a car, v ms^{-1} modelled by $v = 8 - t^3$ for $0 = t = 2$.

7 Velocity of a car, v ms^{-1} modelled by $v = t^2 - 4t + 4$ for $0 = t = 4$.

8 Velocity of a car, v ms^{-1} modelled by $v = 2 - 0.5\sqrt{t}$ for $0 = t = 16$

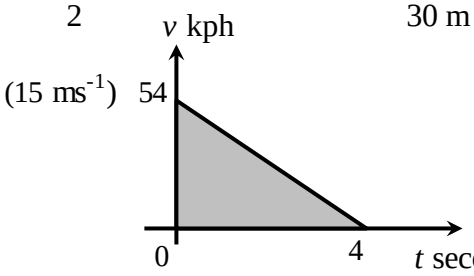
Answers - Page 2

1



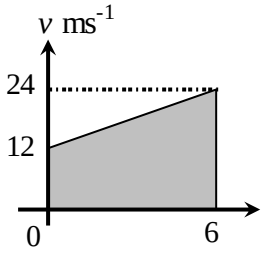
195 miles

2



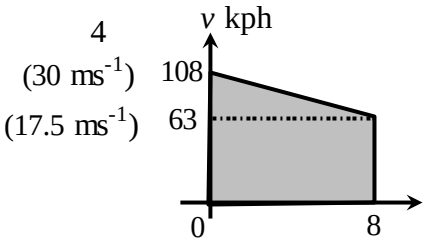
30 m

3



108 m

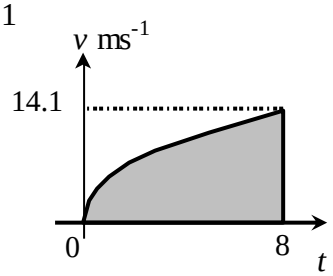
4



190 m

Answers - Page 3

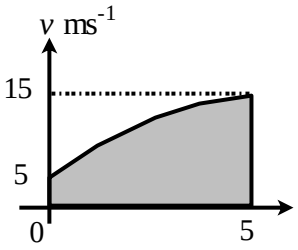
1



Accelerates from
0 ms^{-1} to 14.1 ms^{-1}
in 8 seconds

74 m

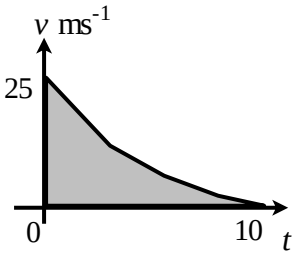
2



Accelerates from
5 ms^{-1} to 15 ms^{-1}
in 5 seconds

58 m

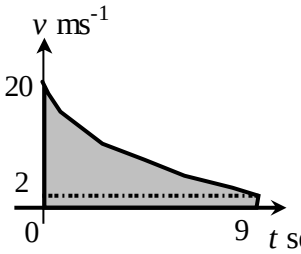
3



Decelerates from
25 ms^{-1} to 0 ms^{-1}
in 10 seconds

85 m

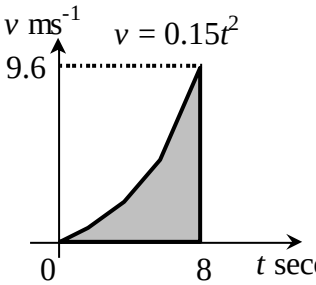
4



Decelerates from
20 ms^{-1} to 2 ms^{-1}
in 9 seconds

73 m

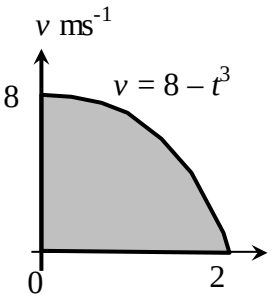
5



Accelerates from
0 ms^{-1} to 9.6 ms^{-1}
in 8 seconds

26 m (1s intervals)

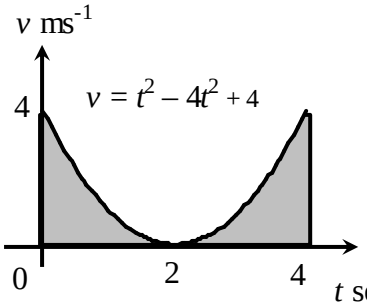
6



Decelerates from
8 ms^{-1} to 0 ms^{-1}
in 2 seconds

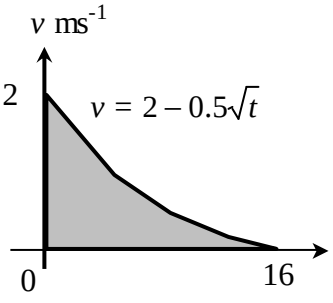
12 m (0.5s intervals)

7



Decelerates from
4 ms^{-1} to 0 ms^{-1}
in 2 seconds, then
accelerates to 4 ms^{-1}
in the next 2 seconds
6 m (1s intervals)

8



Decelerates from
2 ms^{-1} to 0 ms^{-1}
in 16 seconds

10 m (2s intervals)

Teacher Notes

Unit Intermediate Level, Using algebra, functions and graphs

Skills used in this activity:
- finding areas under velocity-time graphs
- estimating areas using triangles and trapezia

Notes on Activity
All the examples on pages 1 to 3 are also included in the Powerpoint presentation of the same name which can be used to introduce this work. Students are expected to know the method for finding the area of a rectangle, but the formulae for the areas of triangles and trapezia are given on page 1. Students can draw the graphs by hand or by using a graphic calculator or spreadsheet.
The answers to the questions, including sketches of the graphs, are given on page 4. More accurate answers to the Page 3 questions could be found by using more strips.