Holiday Money

Four friends saved up to pay for a holiday in Florida in September.
Before going they changed their spending money into dollars.
The bank gave them $1.88 (one dollar and eighty-eight cents) for each pound.
i.e. The exchange rate was £1 = $1.88

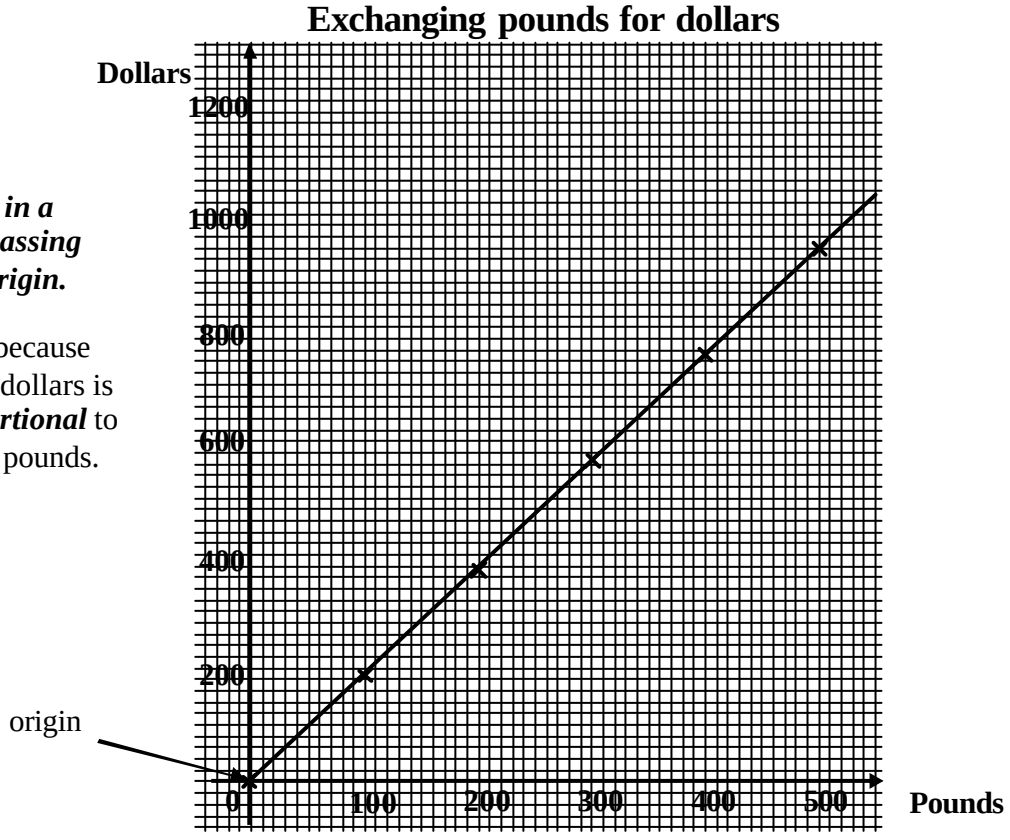
This is how much they had in pounds. How much did they get in dollars?

Sandy £250 Ken £340 Tom £385 Ali £475

The answers could have been worked out from a currency graph. This is how you do it:
First you need a table of values.
Choose easy amounts for the pounds and work out the dollars.

Pounds	0	100	200	300	400	500
Dollars	0	188	376	564	752	940

Draw axes on graph paper.
Use a sensible scale that will give a large graph and be easy to use.
Plot the points and draw a straight line through them.
Here is a sketch of what you should get:



The points lie in a straight line passing through the origin.

This happens because the number of dollars is ***directly proportional*** to the number of pounds.

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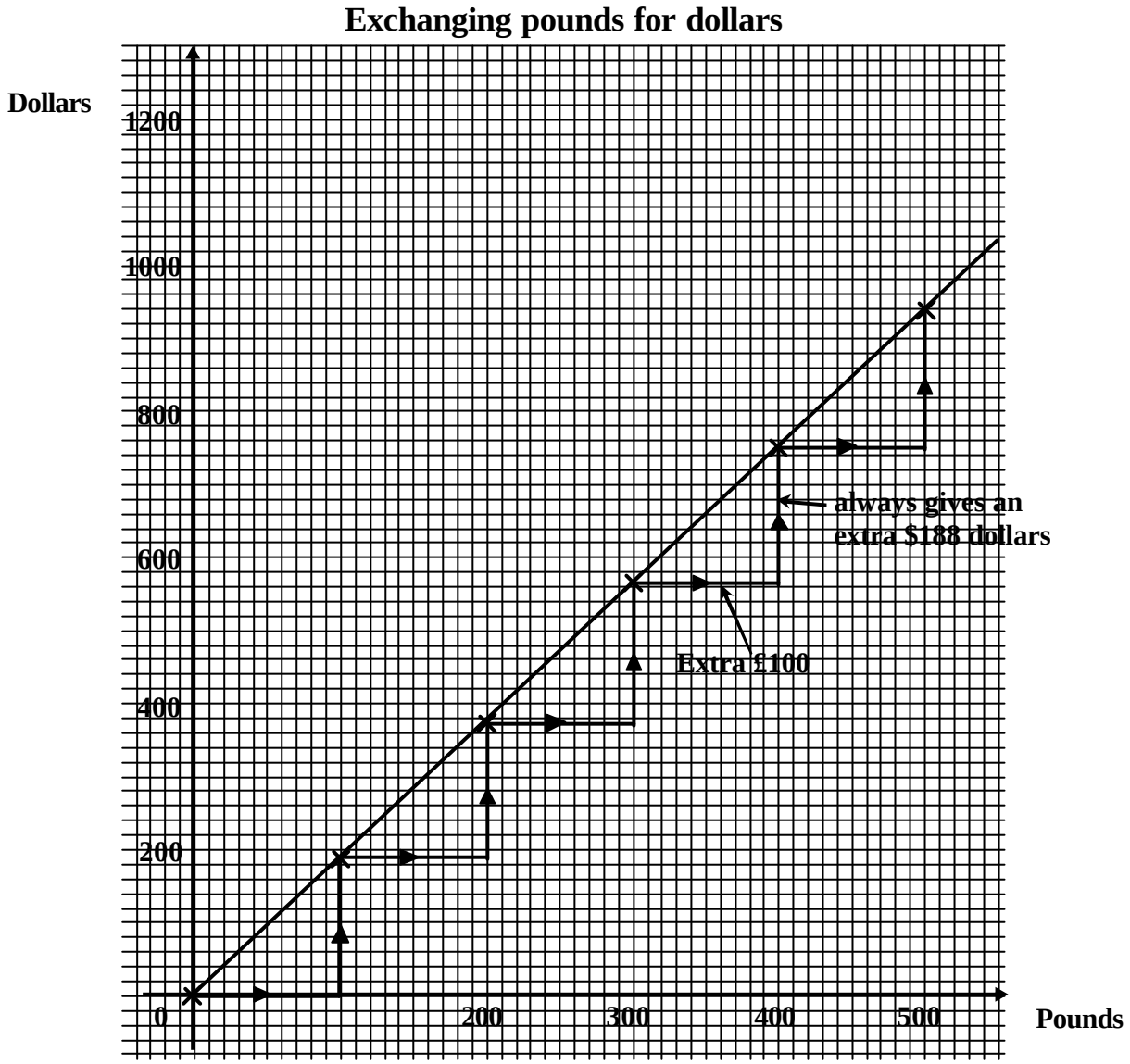
Here is an enlarged graph. It shows that an extra £100 always gives an extra $188.



You can also see this in the table:

Pounds	0	100	200	300	400	500
Dollars	0	188	376	564	752	940

Extra £100

Extra $188

If you double the number of pounds, the number of dollars is also doubled.

Pounds	0	100	200	300	400	500
Dollars	0	188	376	564	752	940

Doubled

Doubled

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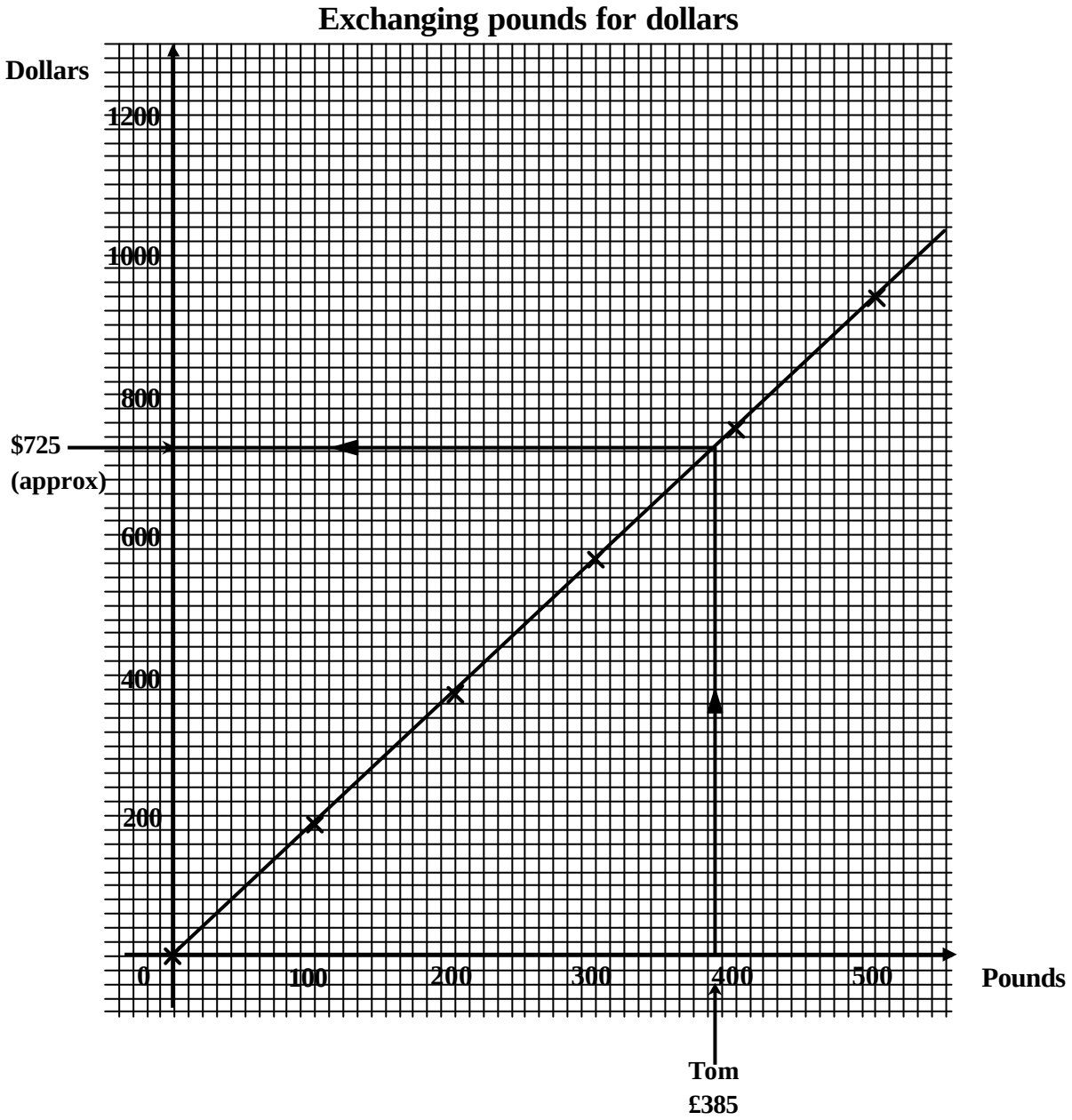
Here is another copy of the graph.



Tom exchanged £385. The graph shows that he got about $725.
The accurate calculation (done earlier) gave $723.80.
The graph gives a good estimate and agrees approximately with the calculation.

Draw lines on the graph to find the number of dollars received by Sandy, Ken and Ali.
The amounts in pounds were Sandy £250 Ken £340 Ali £475

Do the answers from the graph agree with your earlier calculations?

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The holiday was an 'all-inclusive' holiday. The friends did not spend all their money. These are the amounts they had left at the end of the holiday:

Sandy $220 Ken $140 Tom $95 Ali $256

The bank said the exchange rate for returning currency was £1 = $2.15.

Why do you think this is a different rate from the one they got before their holiday?

The new rate can be shown by ***another*** line on the graph. A table is given below.

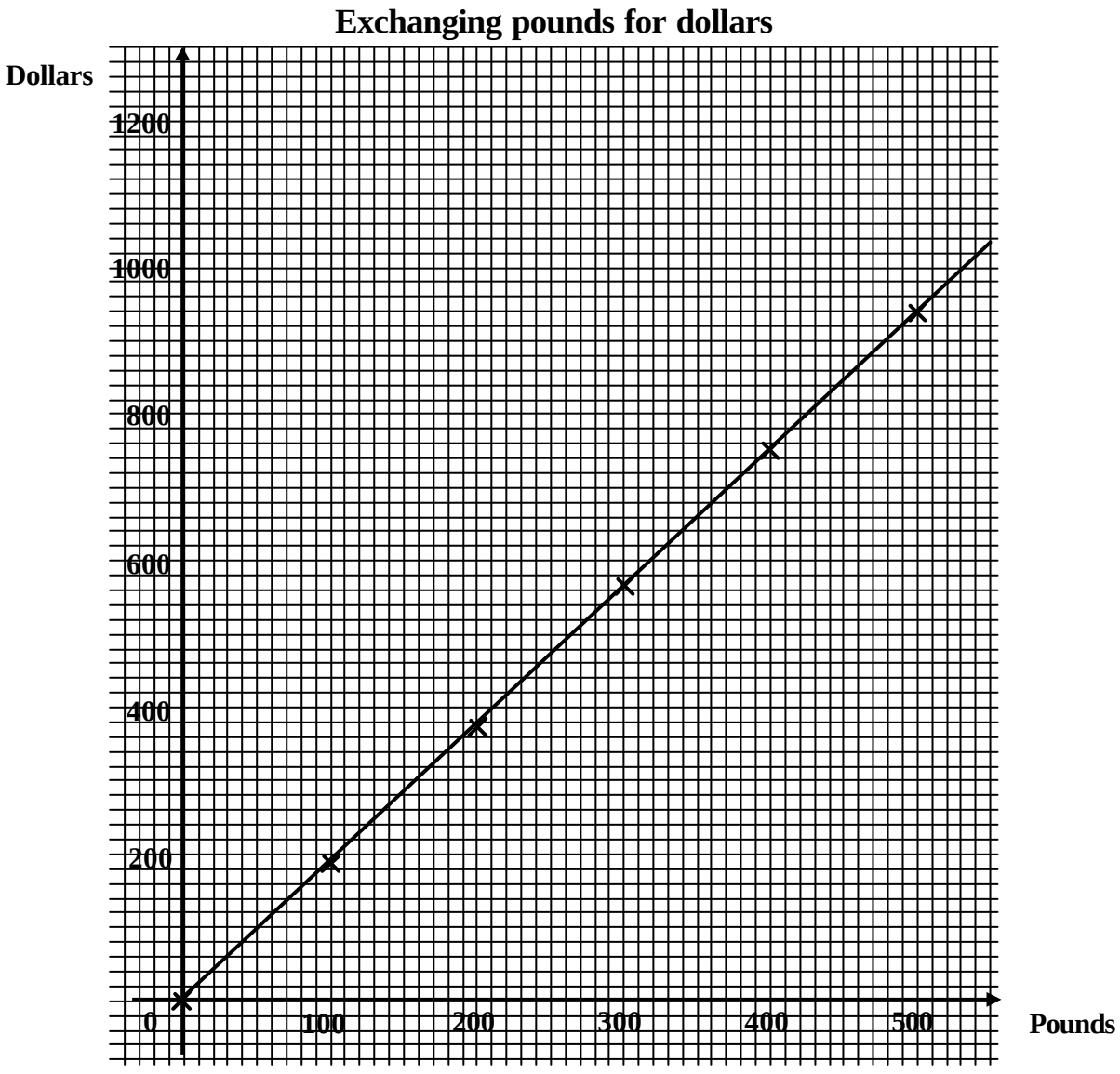
Notice that only 3 amounts have been used. This is the minimum information you should use to draw a straight line graph. (Two points are enough to draw a line – the third point gives a check.)

Complete the table. Draw the line on the graph.

Pounds	0	200	400
Dollars			

Use the new line to find how much each friend got to the nearest pound.

Check your answers by calculation.



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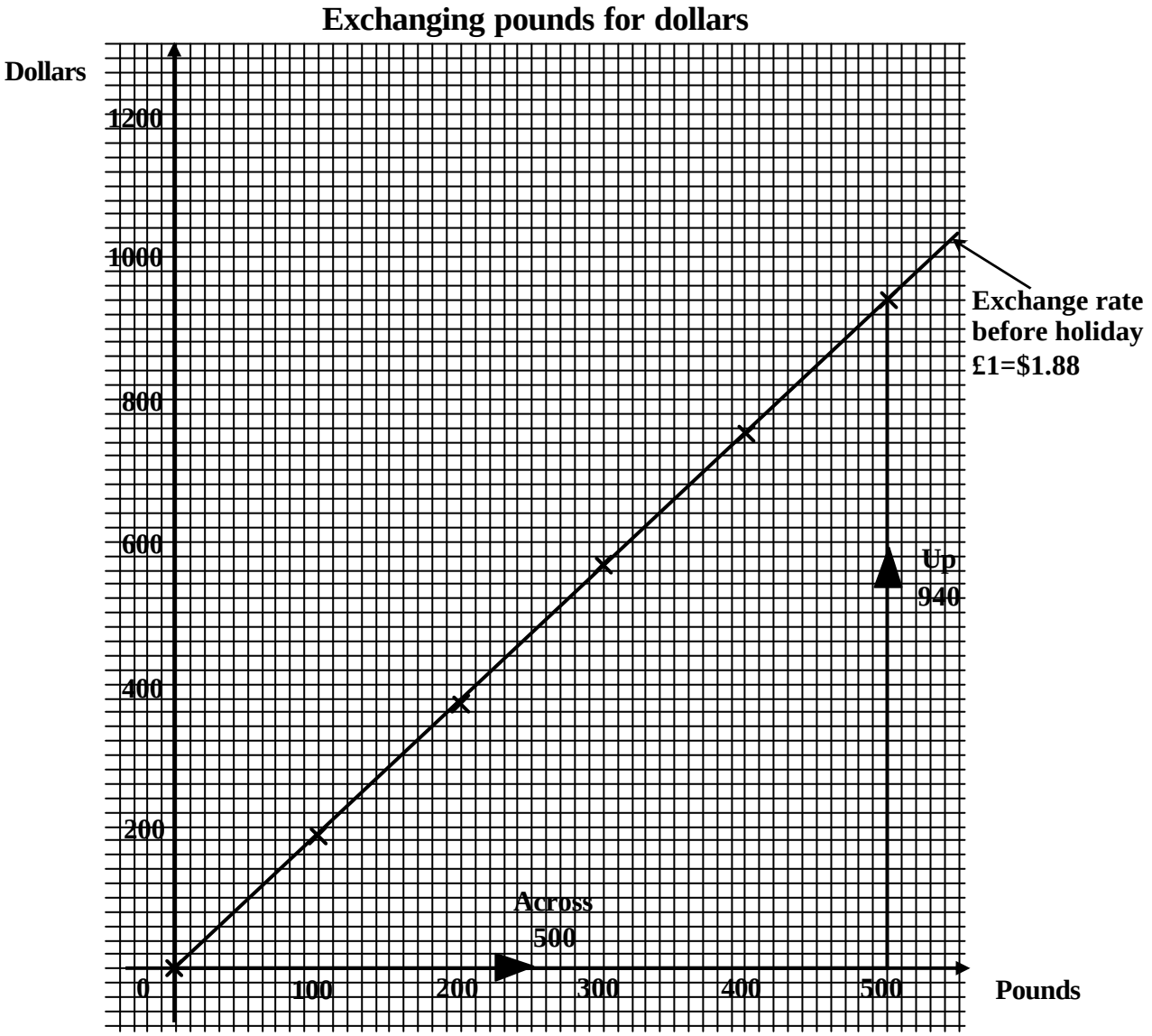
Look again at the two lines on the graph on page 4.
They are ***both straight lines passing through the origin***.

These are the 2 conditions for direct proportionality.

What is different about the two lines? One is steeper than the other. They have different ***gradients***. The gradient of a line can be found by drawing a triangle (the bigger the better) between the origin and a point on the graph as shown below using the first line.



$$\text{Gradient} = \frac{\text{up}}{\text{across}} = \frac{940}{500} = 1.88$$

This is sometimes called ***the constant of proportionality***.
Where have you seen this number before? It is the number in the exchange rate.

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Find the gradient of the second exchange rate line (the one you have drawn on page 4).

A neat way to give the relationship between proportional variables is by using an equation.

When the friends changed their money to go on holiday the bank used the exchange rate £1 = \$1.88.
This could be written as an equation: $D = 1.88P$
D is the number of dollars and P is the number of pounds.

Tom had £385. To find how many dollars he got you could use the equation:

$$\begin{aligned} D &= 1.88P \\ &= 1.88 \times 385 \quad \text{Replace } P \text{ by } 385 \\ &= 723.80 \end{aligned}$$

He got \$723.80.

Use this method to find the number of dollars received by Sandy, Ken and Ali.
The amounts they had in pounds were: Sandy £250 Ken £340 Ali £475

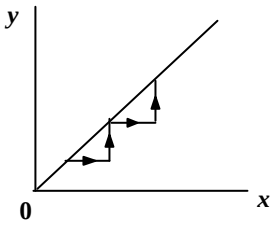
Check your answers with the values you got earlier.

When the friends returned from holiday the bank used the exchange rate £1 = \$2.15
Write this as an equation.
When the friends changed their dollars back to pounds this is what the bank gave them:
Sandy £102 Ken £65 Tom £44 Ali £119

Use your equation to work out how much they had given the bank in dollars.
Check whether your answers agree with the amounts given at the top of page 4.

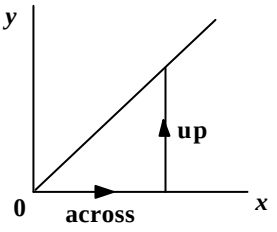
Summary **Proportionality**

When two variables, x and y, are ***directly proportional*** the graph is a ***straight line through the origin***.



Equal increases in x give equal increases in y.
(can be seen in graph or table)
If x is doubled so is y; if x is multiplied by 3 so is y etc.

Gradient of the line = $\frac{\text{up}}{\text{across}}$



x and y are related by the ***equation*** $y = mx$
where m is the gradient (also called the constant of proportionality).

NB Points plotted using ***experimental data*** may be subject to errors in measuring and only ***approximately*** lie on a straight line.

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Direct Proportionality Questions

1 Jen is going on a long journey by motorway. She plans to drive at a steady 70 mph.

a) Copy and complete this table.

Time (hours)	0	3	6
Distance travelled (miles)			

b) Draw a graph with Time on the horizontal axis and Distance on the vertical axis. Use 2 cm to represent 1 hour and 100 miles. Plot the points and draw a straight line.

c) Use the graph to estimate how far Jen will travel in
(i) $2\frac{1}{2}$ hours (ii) $3\frac{1}{4}$ hours (iii) 45 minutes

d) Use the graph to estimate how long it will take Jen to travel
(i) 100 miles (ii) 230 miles (iii) 345 miles

e) If Jen travels at 65 mph instead of 70 mph what difference would this make to the graph you have drawn?

f) In what way is this question unrealistic?

2 A college holds a sponsored collection of litter on Red Nose Day. The people taking part collect as much litter as they can from the college grounds and the amount they have collected is weighed at the end of the day. The principal has offered to sponsor everyone at a rate of 40 pence per kilogram, and the estates manager has offered 25 pence per kilogram.

a) Complete a table like the one below for the amount the principal will pay.

Litter collected (kg)	0	10	20
Amount to pay (£)			

Complete another table showing the amount the estate manager will pay.

b) Draw a graph with Litter collected on the horizontal axis and Amount to pay on the vertical axis. Use the values in your tables to draw two lines.

c) Find the gradient of each line.
Which line is steeper – the principal's or the estate manager's?

d) An equation for working out sponsor money is $A = \ldots . L$
where A represents the amount of sponsor money (in pounds), L is the amount of litter (in kilograms) and …. is a number.
Write down two equations – one for the principal and the other for the estate manager.

e) Use the graph to find how much the principal and estate manager will pay for 13 kg of litter. Check your answers using your equations.

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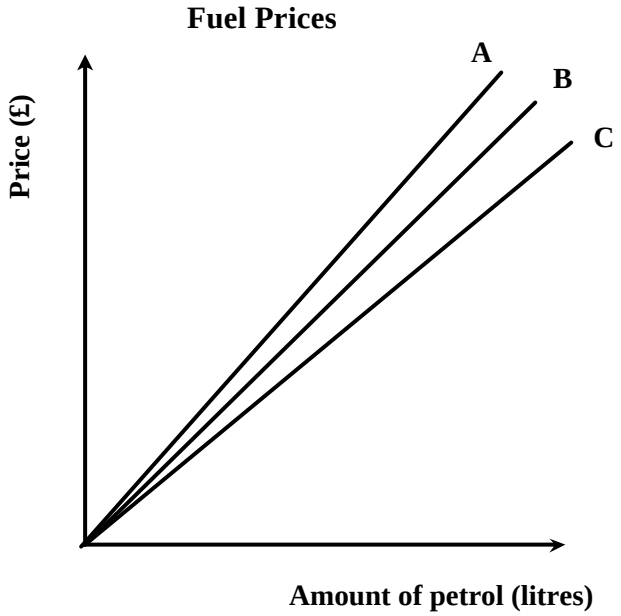
3 Here is a sketch of a graph which can be used to find the prices of different amounts of unleaded petrol, super-unleaded and diesel.



The prices per litre are:

unleaded	£1.12 per litre
super unleaded	£1.20 per litre
diesel	£1.24 per litre

a) State which graph represents each type of petrol.
b) For each type of fuel write down an equation which can be used to work out the prices of different amounts.
c) Use your equations to find the cost of 40 litres of each type of fuel.

4 Here are some tables of prices.
In which cases is the price directly proportional to the weight?

a) Potatoes

Weight (pounds)	5	10	20
Price	£1.80	£3.60	£7.20

b) Parcel Post

Weight (kg)	2	4	6
Price	£4.20	£6.85	£9.30

c) Top Soil (excluding delivery)

Weight (tons)	1	3	5
Price	£90	£270	£450

d) Coffee

Weight (grams)	100	200	300
Price	£1.94	£3.78	£5.54

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5 A student has a collection of 2p coins. In an experiment she puts these coins into piles and measures the height of the piles. Here are the results:

Number of coins	5	10	15	20	25
Height of pile (mm)	10	20	30	40	50

a) Draw a graph of these results using number of coins on the horizontal axis.
b) Is the height of the pile directly proportional to the number of coins?
Explain your answer.
c) Find an equation relating the height of the pile and the number of coins.
d) Use your equation to estimate the height of a pile of 18 coins.
Use your graph to check your answer.
e) Estimate the height of a pile of 65 coins.
f) If the student had used 50p coins instead of 2p coins in her experiment what difference would this have made to the graph?

Experiments

Choose one of the following or think of another situation with two variables.
Plan and carry out an experiment to see whether the variables are directly proportional.
(Remember not to expect an exact straight line from experimental results.)
If the variables are directly proportional, write down an equation which relates them.

1 Number of coins and their weight.

2 Time to burn a candle and the reduction in its length.

3 Weights attached to a piece of elastic and the increase in its length.

4 Amount of water in a kettle and the time it takes to boil.

5 Diameter of cans and their circumference.

6 Distance walked and the time taken.

7 Number of lengths and the time taken to swim them.

8 Number of potatoes and the time taken to peel them.

9 Time after pulling out the plug and the decrease in depth of the water in the bath.

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Teacher Notes

Unit Foundation Level, Making sense of data

Skills used in this activity:

- calculating values in situations involving direct proportionality
- drawing graphs and finding gradients
- finding the equation relating proportional variables

Preparation For the class you need:

- work sheets (pages 1 - 6) or OHPs covering the same work (see notes below)
- a copy of the exercise (pages 7 and 8) for each student
- graph paper
- equipment for experiments (if done in class)

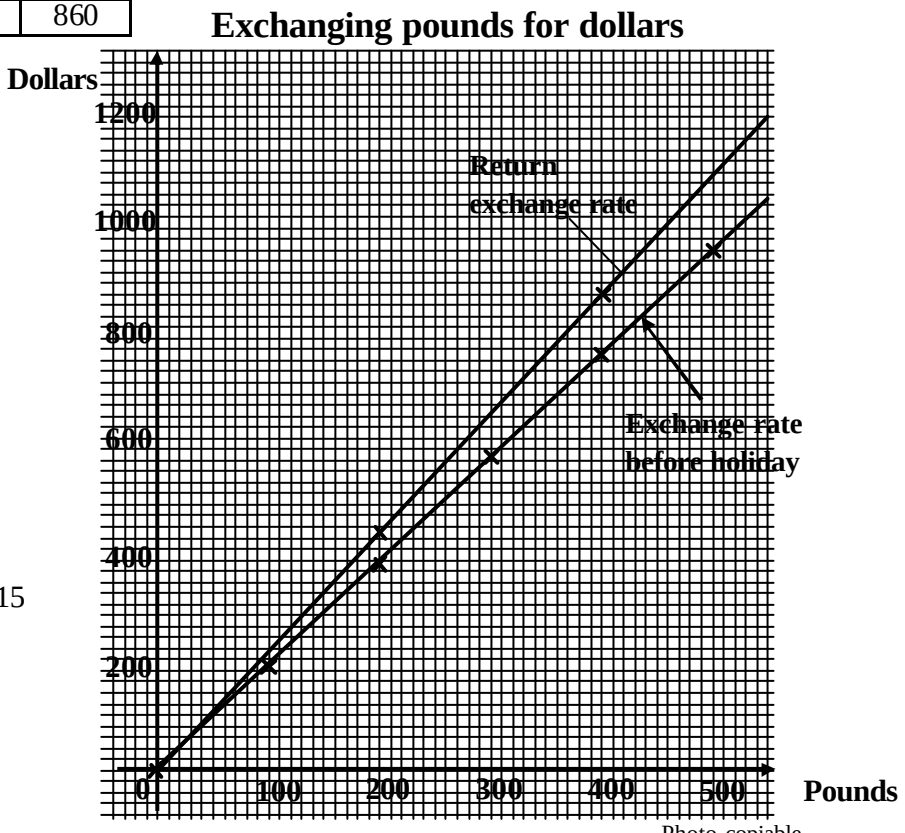
Notes on the Activity

The worksheets can be used for students working independently.
Alternatively pages 12, 13 and 14 can be shown on an Activboard or copied onto transparencies to be used in class teaching.
Answers for questions on the worksheets are given below.

Page 1, 3 and 6 Sandy $470 Ken $639.20 Tom $723.80 Ali $893

Page 4

Pounds	0	100	200
Dollars	0	430	860



Page 4 and 6
At end of holiday amounts received from bank in return for remaining dollars are:

Sandy £102
Ken £65
Tom £44
Ali £119

Page 6
Gradient of second exchange rate line is 2.15
Equation $D = 2.15P$

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Answers to Direct Proportionality Questions

NB Answers are accurate to the number of figures given.
Allow some leeway for answers found from graphs.

1 a)

Time (hours)	0	3	6
Distance travelled (miles)	0	210	420

c) (i) 175 miles (ii) 228 miles (iii) 53 miles
d) (i) 1.43 hours (or 1 hour 26 mins) (ii) 3.29 hours (or 3 hours 17 mins)
(iii) 4.93 hours (or 4 h 56 mins)
e) less steep gradient
f) unlikely to be able to maintain a constant speed on a motorway

2 a) Principal

Litter collected (kg)	0	10	20
Amount to pay (£)	0	4	8

Estate manager

Litter collected (kg)	0	10	20
Amount to pay (£)	0	2.5	5

c) Principal 0.4 Estate manager 0.25 Principal's line is steeper.
d Principal $A = 0.4L$ Estate manager $A = 0.25L$
e) Principal £5.20 Estate manager £3.25

3 a) unleaded C super unleaded B diesel A
b) unleaded $P = 1.12A$ super unleaded $P = 1.2A$ diesel $P = 1.24A$
where P is the price in £ and A the amount in litres
c) unleaded £44.80 super unleaded £48 diesel £49.60

4 a) and c)

5 b) Yes – straight line passing through O.
c) $h = 2n$ where h is the height in mm and n the number of coins
d) 36 mm or 3.6 cm
e) 130 mm or 13 cm
f) different (steeper) gradient because of different thickness

Experime nts

These could be used to generate evidence for the students' Coursework Portfolio. Some of these could be done in class. For others it would be necessary for students to collect the necessary data outside class.

N.B. Wherever possible experiments which are relevant to students' other studies or interests should be used instead.

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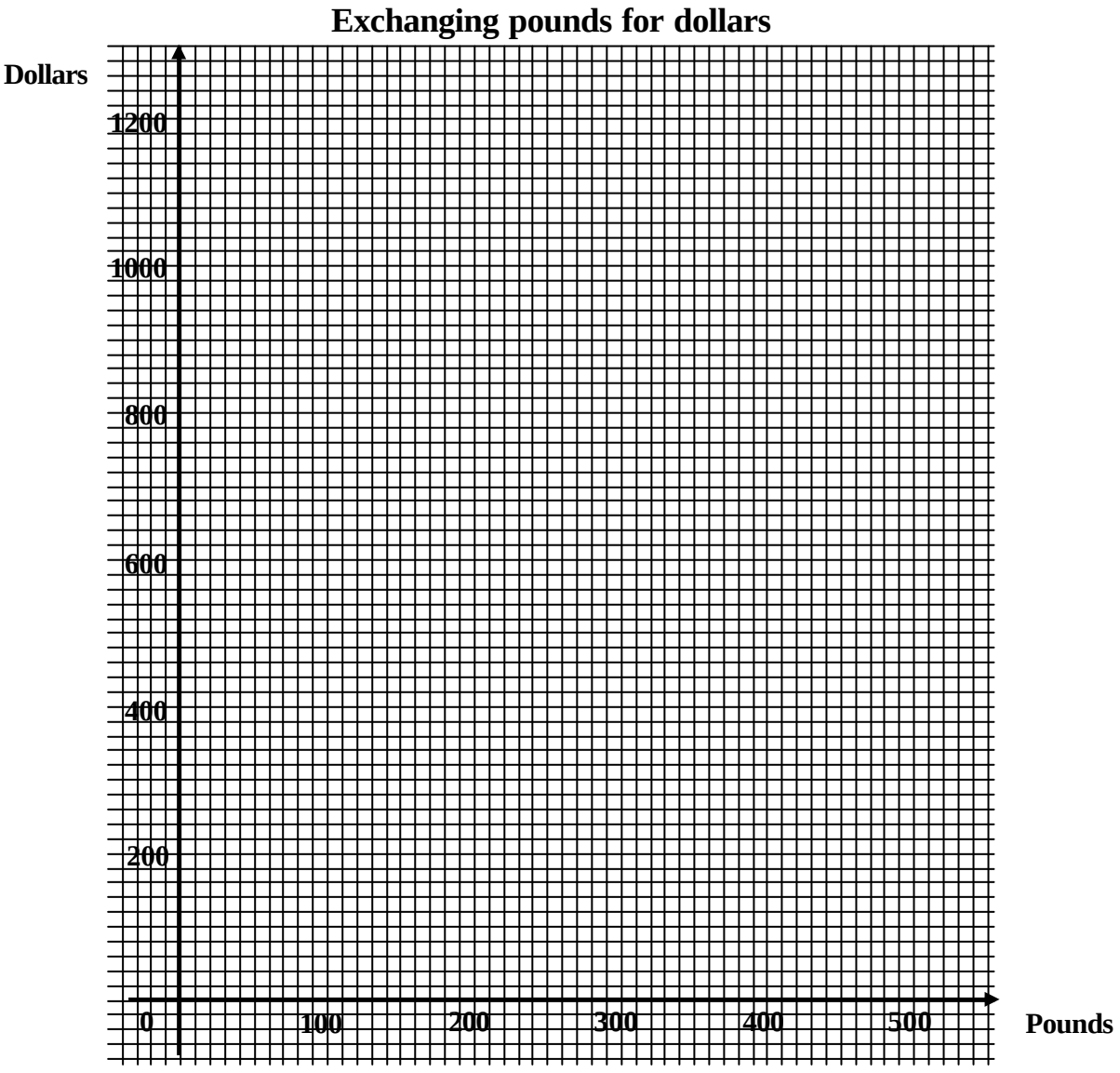
Holiday Money

Pounds					
Dollars					



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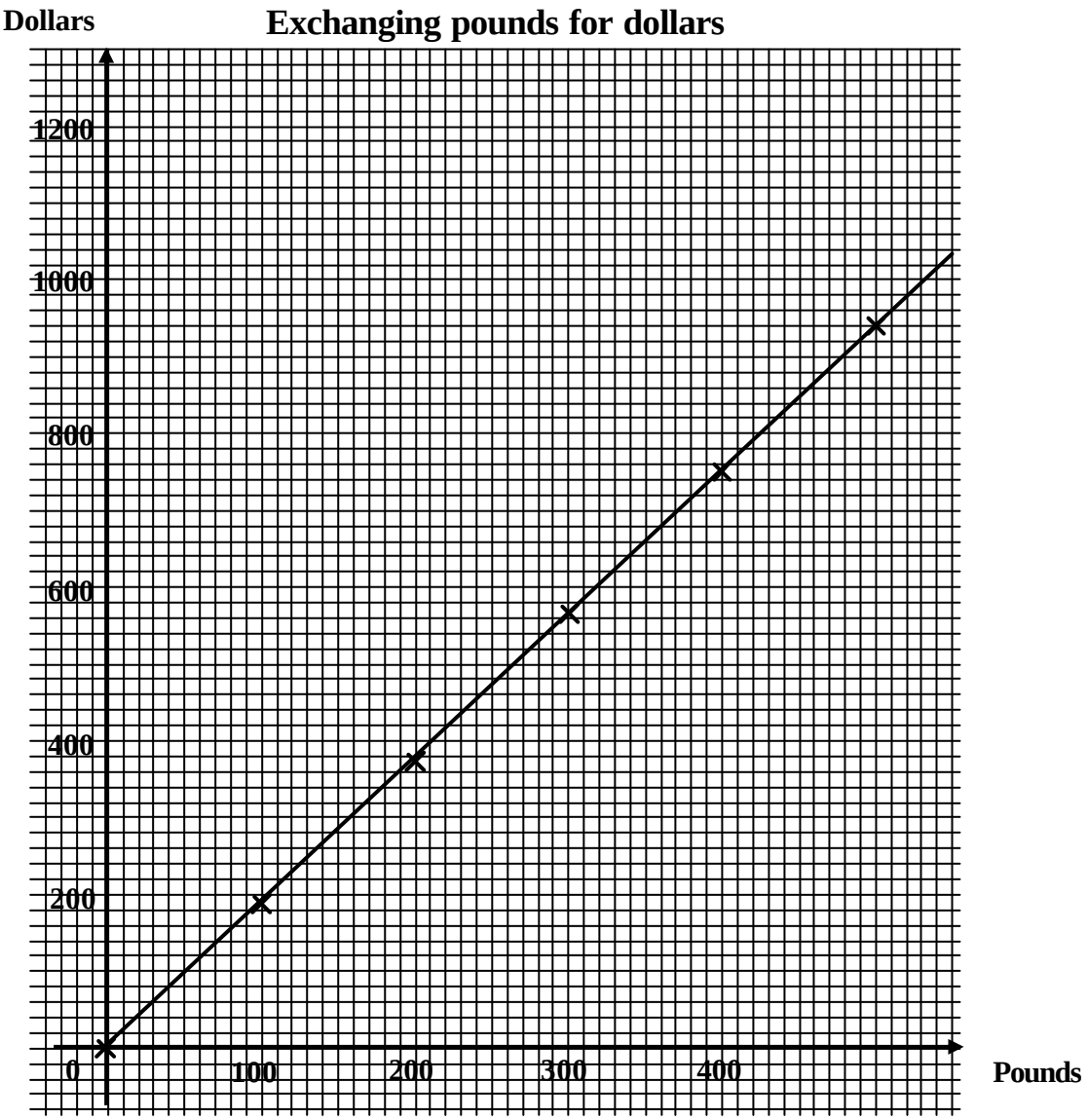
Holiday Money

Pounds	0	100	200	300	400	500
Dollars	0	188	376	564	752	940



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Holiday Money

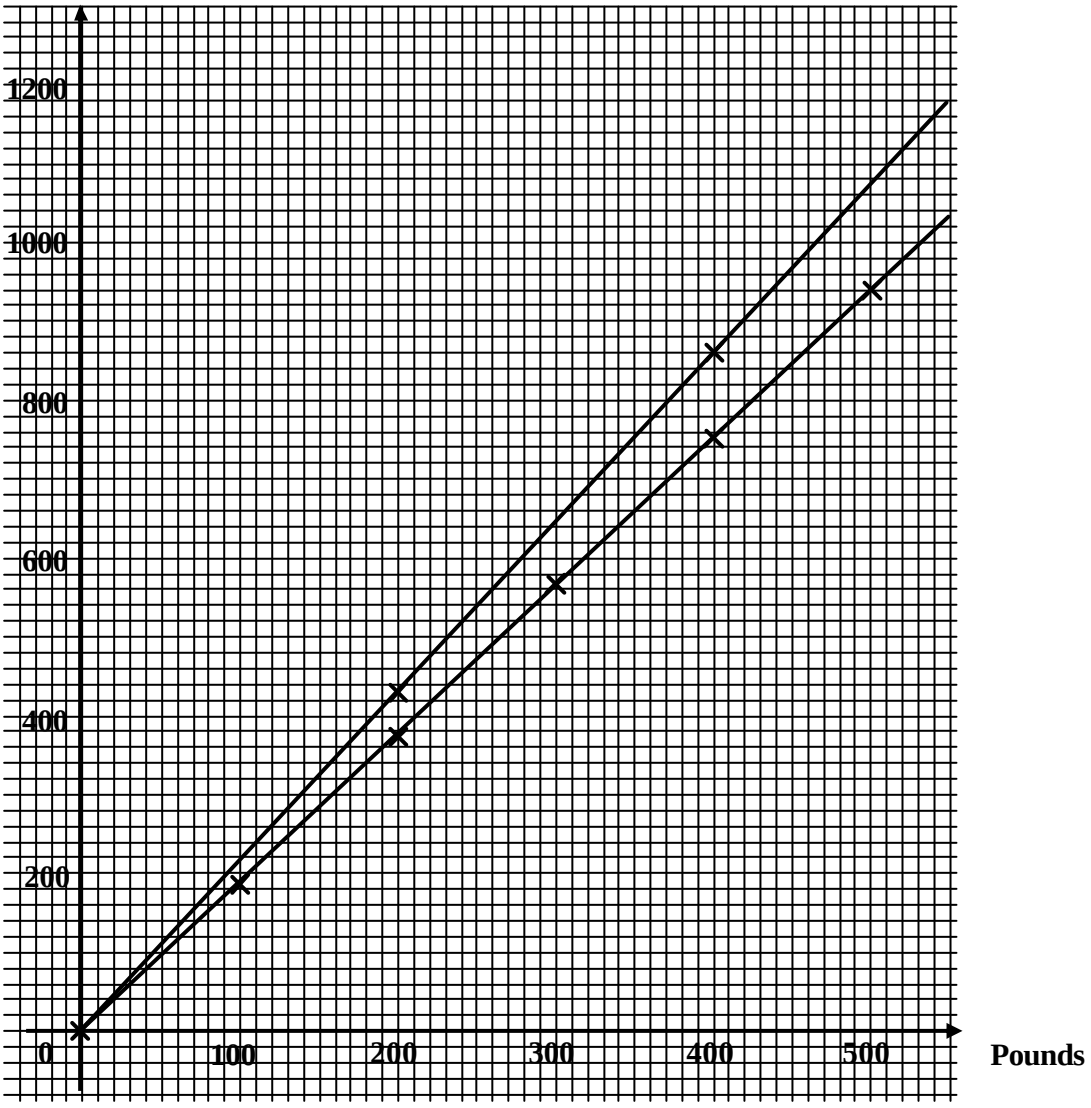
£1 = $1.88

Pounds	0	100	200	300	400	500
Dollars	0	188	376	564	752	940

£1 = $2.15

Pounds	0	100	200
Dollars	0	215	430

Exchanging pounds for dollars



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