

Foreign Currency



Four friends worked all summer to pay for a holiday in Florida in September. Before going they changed their spending money into dollars. The bank gave them \$1.55 (one dollar and fifty-five cents) for each pound. i.e. The exchange rate was £1 = \$1.55

This is how much they had in pounds. How much did they get in dollars?

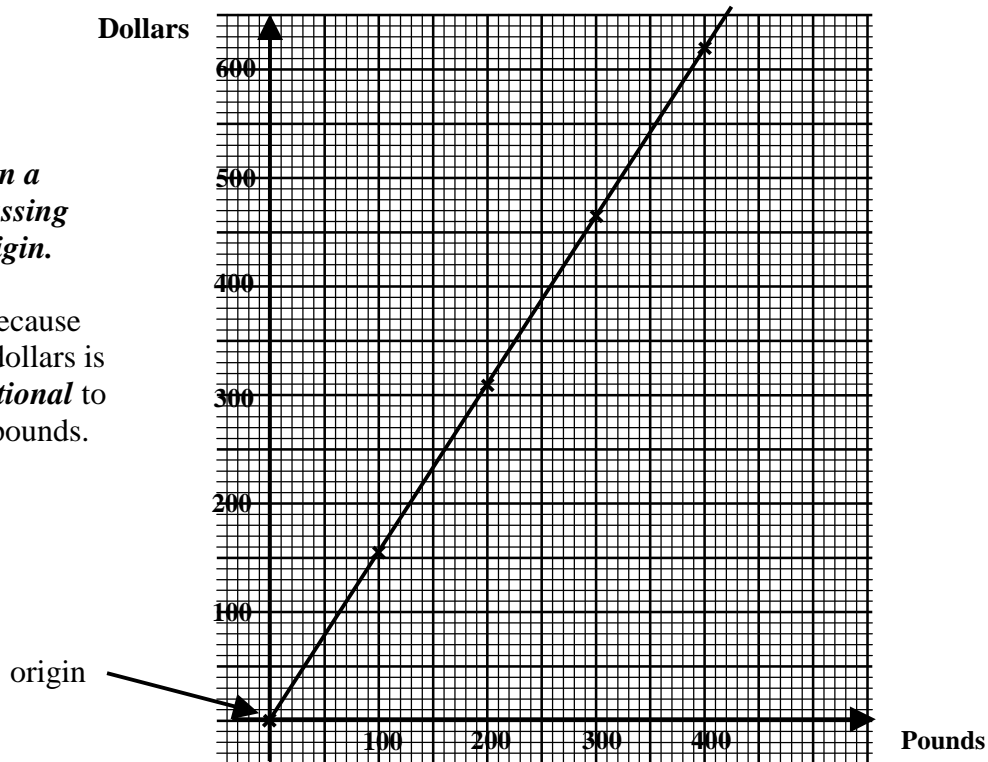
Sandy £350 Ken £290 Tom £275 Ali £315

The answers could have been worked out from a currency graph. This is how you do it: First you need a table of values. Choose easy amounts for the pounds and work out the dollars.

Pounds	0	100	200	300	400
Dollars	0	155	310	465	620

Draw axes on graph paper. Use a sensible scale that will give a large graph and be easy to use. Plot the points and draw a straight line through them. Here is a sketch of what you get:

Exchanging pounds for dollars



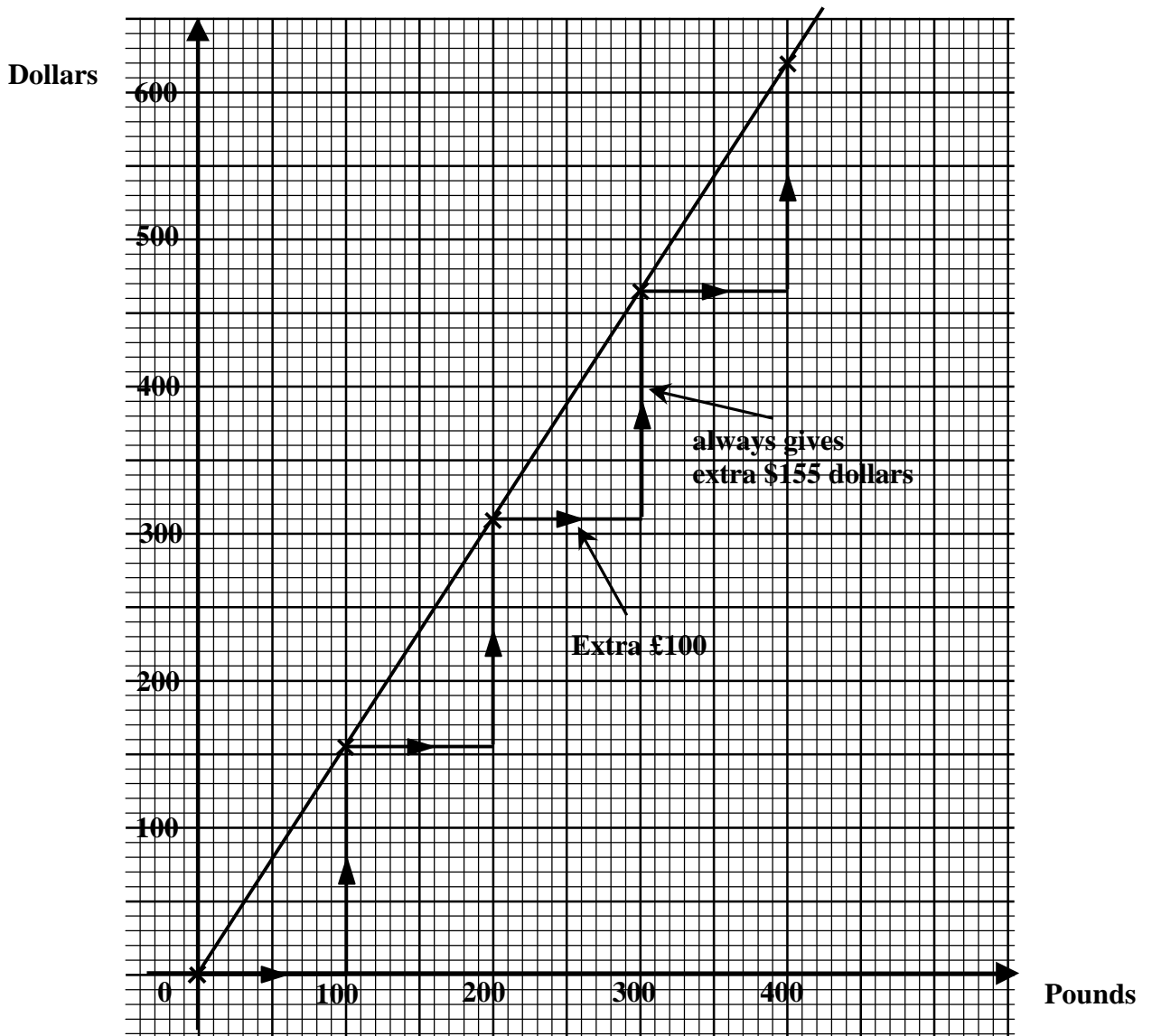
The points lie in a straight line passing through the origin.

This happens because the number of dollars is *directly proportional* to the number of pounds.



Here is an enlarged graph. It shows that an extra £100 always gives an extra \$155.

Exchanging pounds for dollars



You can also see this in the table:

Pounds	0	100	200	300	400
Dollars	0	155	310	465	620

Extra £100

Extra \$155

If you double the number of pounds, the number of dollars is also doubled.

Pounds	0	100	200	300	400
Dollars	0	155	310	465	620

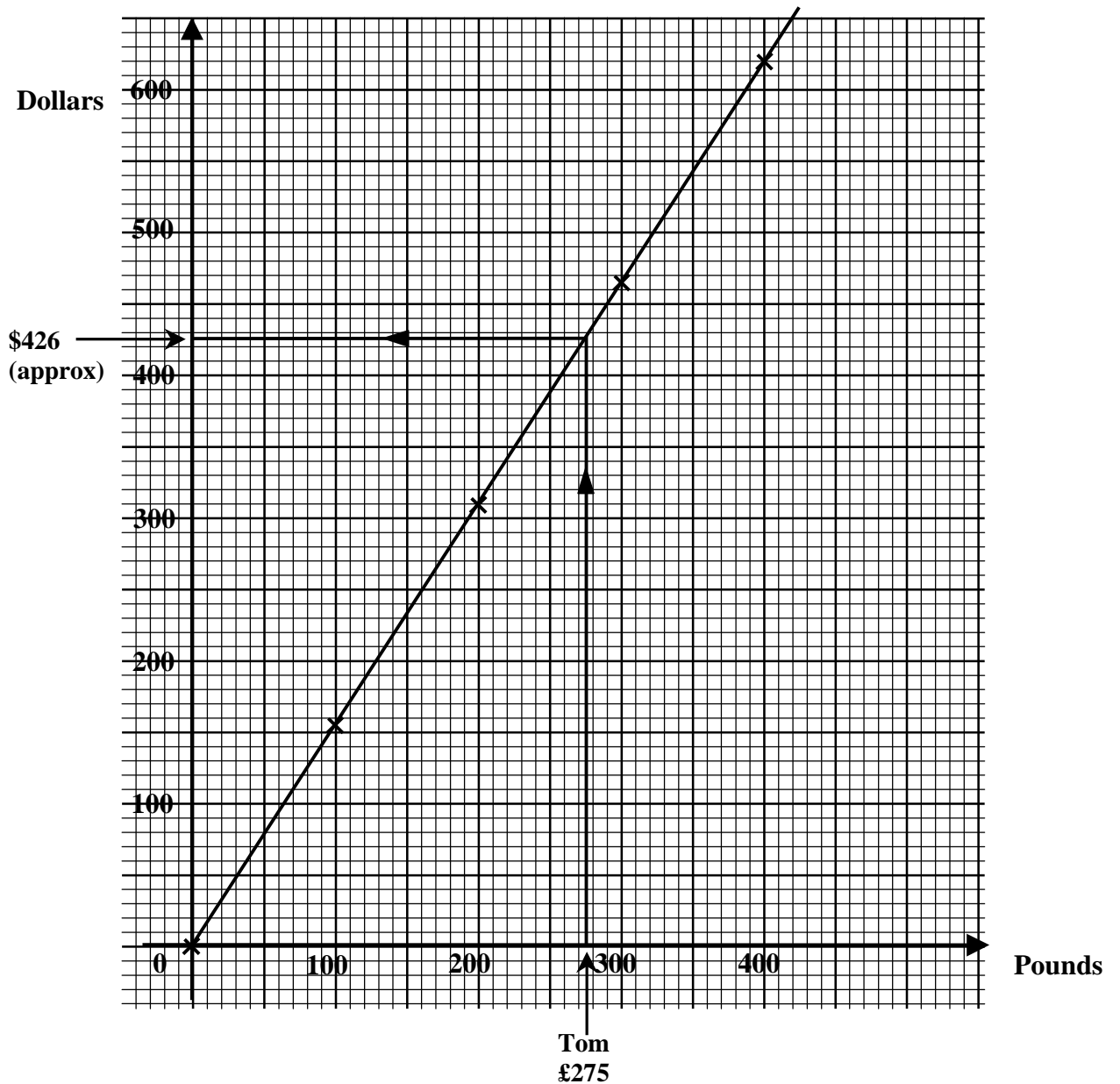
Doubled

Doubled



Here is another copy of the graph.

Exchanging pounds for dollars



Tom exchanged £275. The graph shows that he got approximately \$426.
 The accurate calculation (done earlier) gave \$426.25.
 The graph gives a good estimate and agrees approximately with the calculation.

Draw lines on the graph to find the number of dollars received by Sandy, Ken and Ali.
 The amounts in pounds were Sandy £350 Ken £290 Ali £315

Do the answers from the graph agree with your earlier calculations?



The holiday was an ‘all-inclusive’ holiday. The friends did not need to spend much. At the end of the holiday they had a lot of money left. These are the amounts:

Sandy \$220 Ken \$140 Tom \$85 Ali \$264

The bank said the exchange rate for returning currency was £1 = \$1.70.

Why do you think this is a different rate from the one they got before their holiday?

The new rate can be shown by *another* line on the graph. A table is given below.

Notice that only 3 amounts have been used. This is the minimum information you should use to draw a straight line graph. (Two points are enough to draw a line – the third point gives a check.)

Complete the table. Draw the line on the graph.

Pounds	0	100	200
Dollars			

Use the new line to find how much each friend got in pounds. (N.B. Start on the dollars axis.)

Check your answers by calculation.

Exchanging pounds for dollars

Dollars

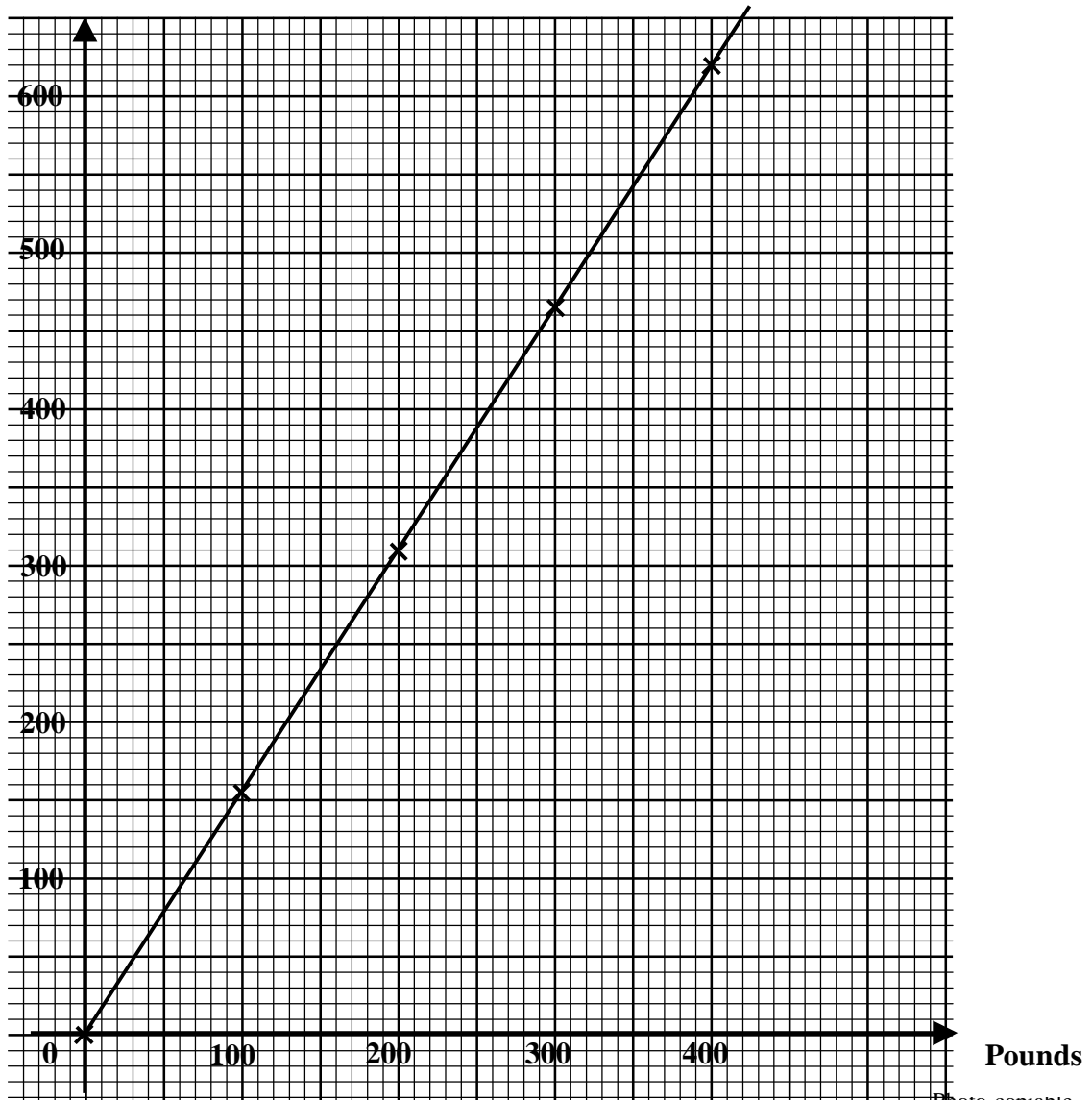


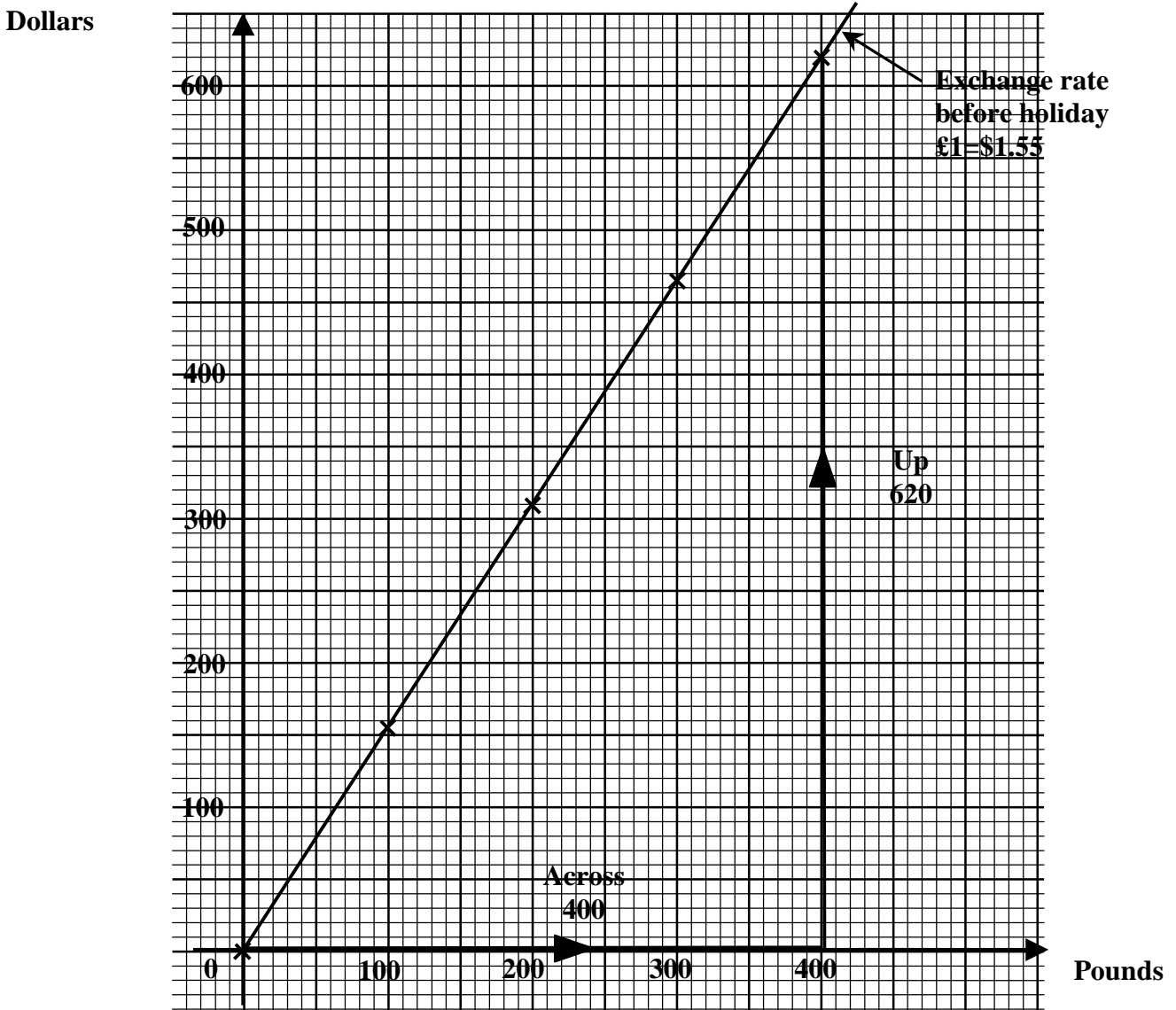
Photo-copiable



Look again at the two lines on the graph on page 4. They are *both straight lines passing through the origin*. ← *These are the 2 conditions for direct proportionality.*

What is different about the two lines? One is steeper than the other. They have different *gradients*. The gradient of a line can be found by drawing a triangle (the bigger the better) between the origin and a point on the graph as shown below using the first line.

Exchanging pounds for dollars



$$\text{Gradient} = \frac{\text{up}}{\text{across}} = \frac{620}{400} = 1.55$$

This is sometimes called *the constant of proportionality*. Where have you seen this number before? It is the number in the exchange rate.



Find the gradient of the second exchange rate line (the one you have drawn on page 4).

A neat way to give the relationship between proportional variables is by using an equation.

When the friends changed their money to go on holiday the bank used the exchange rate £1 = \$1.55.

This could be written as an equation: $D = 1.55P$

D is the number of dollars and P is the number of pounds.

Tom had £275. To find how many dollars he got you could use the equation:

$$\begin{aligned} D &= 1.55P \\ &= 1.55 \times 275 \quad \leftarrow \text{Replace } P \text{ by } 275 \\ &= 426.25 \end{aligned}$$

He got \$426.25.

Use this method to find the number of dollars received by Sandy, Ken and Ali.

The amounts they had in pounds were: Sandy £350 Ken £290 Ali £315

Check your answers with the values you got earlier.

When the friends returned from holiday the bank used the exchange rate £1 = \$1.70

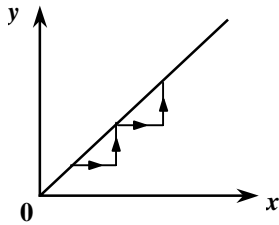
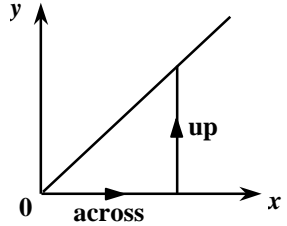
Write this as an equation.

When the friends changed their dollars back to pounds this is what the bank gave them:

Sandy £129.41 Ken £82.35 Tom £50 Ali £155.29

Use your equation to work out how much they had given the bank in dollars.

Check your answers agree with the amounts given at the top of page 4.

Summary	Proportionality
When two variables, x and y , are <i>directly proportional</i> the graph is a <i>straight line through the origin</i> .	
Equal increases in x give equal increases in y . (can be seen in graph or table)	
If x is doubled so is y ; if x is multiplied by 3 so is y etc.	
Gradient of the line = $\frac{\text{up}}{\text{across}}$	
x and y are related by the equation $y = mx$ where m is the gradient (also called the constant of proportionality).	
NB Points plotted using <i>experimental data</i> may be subject to errors in measuring and only <i>approximately</i> lie on a straight line.	



Direct Proportionality Questions

- 1 Jen is going on a long journey by motorway. She plans to drive at a steady 70 mph.

a) Copy and complete this table.

Time (hours)	0	3	6
Distance travelled (miles)			

- b) Draw a graph with Time on the horizontal axis and Distance on the vertical axis. Use 2 cm to represent 1 hour and 100 miles. Plot the points and draw a straight line.
- c) Use the graph to estimate how far Jen will travel in
 (i) $2\frac{1}{2}$ hours (ii) $3\frac{1}{4}$ hours (iii) 45 minutes
- d) Use the graph to estimate how long it will take Jen to travel
 (i) 100 miles (ii) 230 miles (iii) 345 miles
- e) If Jen travels at 65 mph instead of 70 mph what difference would this make to the graph you have drawn?
- f) In what way is this question unrealistic?

- 2 A college holds a sponsored collection of litter on Red Nose Day. The people taking part collect as much litter as they can from the college grounds and the amount they have collected is weighed at the end of the day. The principal has offered to sponsor everyone at a rate of 40 pence per kilogram, and the estates manager has offered 25 pence per kilogram.

a) Complete a table like the one below for the amount the principal will pay.

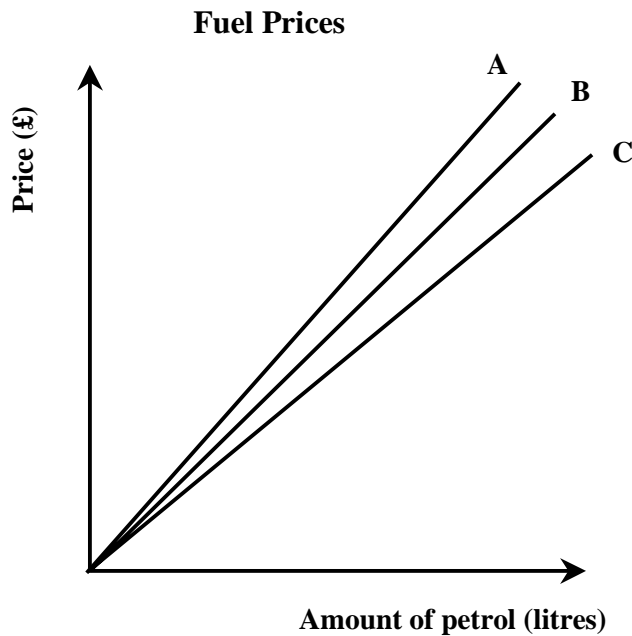
Litter collected (kg)	0	10	20
Amount to pay (£)			

Complete another table showing the amount the estate manager will pay.

- b) Draw a graph with Litter collected on the horizontal axis and Amount to pay on the vertical axis. Use the values in your tables to draw two lines.
- c) Find the gradient of each line.
Which line is steeper – the principal's or the estate manager's?
- d) An equation for working out sponsor money is $A = \dots L$
where A represents the amount of sponsor money (in pounds), L is the amount of litter (in kilograms) and \dots is a number.
Write down two equations – one for the principal and the other for the estate manager.
- e) Use the graph to find how much the principal and estate manager will pay for 13 kg of litter. Check your answers using your equations.



- 3 Here is a sketch of a graph which can be used to find the prices of different amounts of unleaded petrol, lead replacement petrol and diesel.



The prices per litre are:

unleaded 80.9 pence per litre
 LRP 86.9 pence per litre
 diesel 81.9 pence per litre

- a) State which graph represents each type of petrol.
 b) For each type of fuel write down an equation which can be used to work out the prices of different amounts.
 c) Use your equations to find the cost of 40 litres of each type of fuel.
- 4 Here are some tables of prices.
 In which cases is the price directly proportional to the weight?

a) **Potatoes**

Weight (pounds)	5	10	20
Price	£0.60	£1.20	£2.40

b) **Parcel Post**

Weight (kg)	5	10	15
Price	£6.25	£7.95	£9.30

c) **Top Soil (excluding delivery)**

Weight (tons)	1	3	5
Price	£30	£90	£150

d) **Coffee**

Weight (grams)	100	200	300
Price	£1.90	£3.40	£4.90

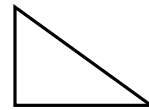
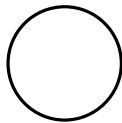
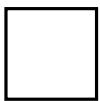


- 5 A student has a collection of 2p coins. In an experiment she puts these coins into piles and measures the height of the piles. Here are the results:

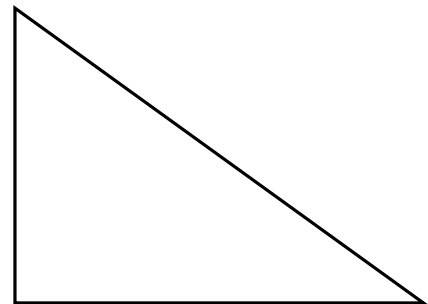
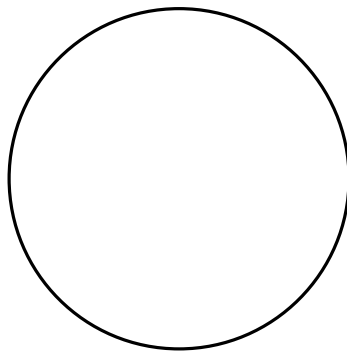
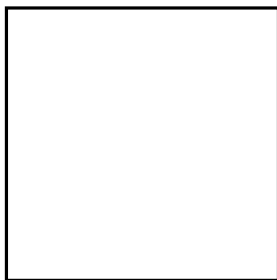
Number of coins	5	10	15	20	25
Height of pile (mm)	10	20	30	40	50

- Draw a graph of these results using number of coins on the horizontal axis.
 - Is the height of the pile directly proportional to the number of coins?
Explain your answer.
 - Find an equation relating the height of the pile and the number of coins.
 - Use your equation to estimate the height of a pile of 18 coins.
Use your graph to check your answer.
 - Estimate the height of a pile of 65 coins.
 - If the student had used 50p coins instead of 2p coins in her experiment what difference would this have made to the graph?
- 6 These shapes have all been enlarged using the same scale factor.

Original Shapes



Enlargements



- a) Copy and complete this table by measuring the features required:

Shape	Feature	Length in Original Shape (x)	Length in Enlargement (y)
Square	Side		
Circle	Diameter		
Triangle	Base		
Triangle	Height		

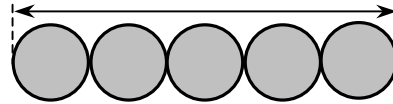
- What is the scale factor of the enlargement?
- Draw a graph and plot the points given in your table.
- Find the gradient of the graph and write down an equation relating y and x .



Experiments

Choose one of the following or think of another situation with two variables. Plan and carry out an experiment to see whether the variables are directly proportional. (Remember not to expect an exact straight line from experimental results.) If the variables are directly proportional, write down an equation which relates them.

- 1 Number of coins in a row and the distance from one end of the row of coins to the other.



- 2 Time to burn a candle and the reduction in its length.
- 3 Weights attached to a piece of elastic and the increase in its length.
- 4 Amount of water in a kettle and the time it takes to boil.
- 5 Diameter of cans and their circumference.
- 6 Distance walked and the time taken.
- 7 Number of lengths and the time taken to swim them.
- 8 Number of potatoes and the time taken to peel them.
- 9 Time after pulling out the plug and the decrease in depth of the water in the bath.



Teacher Notes

Unit Intermediate Level, Making connections in mathematics

Skills used in this activity:

- calculating values in situations involving direct proportionality
- drawing graphs and finding gradients
- finding the equation relating proportional variables

Preparation For the class you need:

- worksheets (pages 1 - 6) or OHPs covering the same work (see notes below)
- a copy of the exercise (pages 7 - 9) for each student
- graph paper
- equipment for experiments if done in class (for experiment ideas see page 10)

Notes on the Activity

The worksheets can be used for students working independently. Alternatively pages 14, 15 and 16 can be copied on OHPs and used in class discussion. Answers for questions on the worksheets are given below.

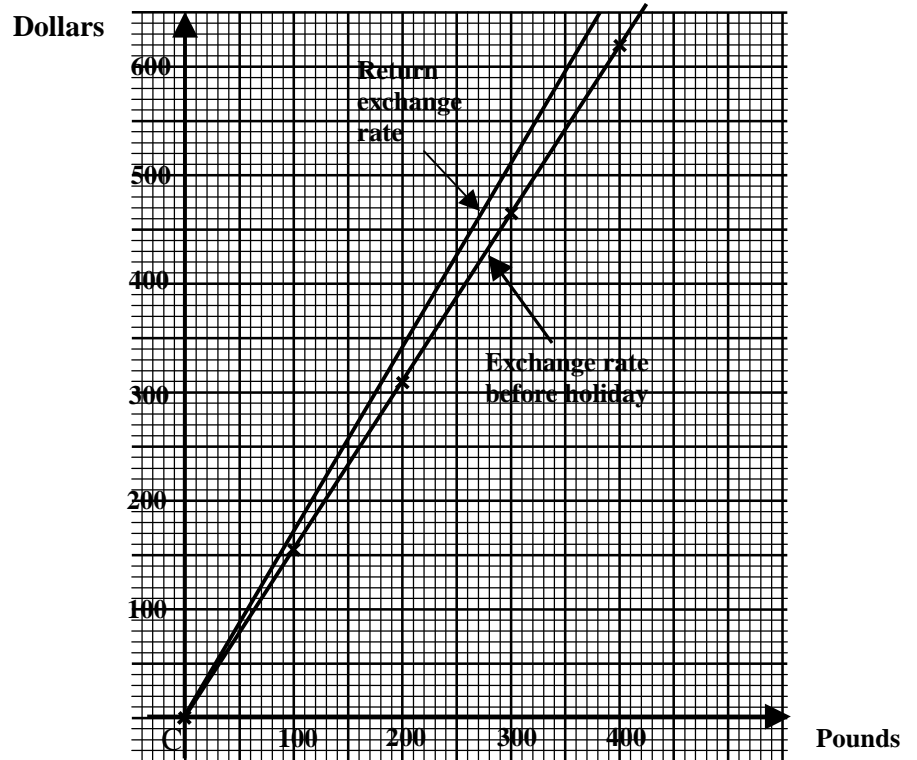
Page 1, 3 and 6 Sandy \$542.50 Ken \$449.50 Tom \$426.25 Ali \$488.25
 Page 4

Pounds	0	100	200
Dollars	0	170	340

Page 4 and 6
 At end of holiday amounts received from bank in return for remaining dollars are:
 Sandy £129.41
 Ken £82.35
 Tom £50
 Ali £155.29

Page 6
 Gradient of second exchange rate line is 1.7
 Equation $D = 1.7P$

Exchanging pounds for dollars



Answers to Direct Proportionality Questions

NB Answers are accurate to the number of figures given.
Allow some leeway for answers found from graphs.

1 a)

Time (hours)	0	3	6
Distance travelled (miles)	0	210	420

- c) (i) 175 miles (ii) 228 miles (iii) 53 miles
 d) (i) 1.43 hours (or 1 hour 26 mins) (ii) 3.29 hours (or 3 hours 17 mins)
 (iii) 4.93 hours (or 4 h 56 mins)
 e) less steep gradient
 f) unlikely to be able to maintain a constant speed on a motorway

2 a) Principal

Litter collected (kg)	0	10	20
Amount to pay (£)	0	4	8

Estate manager

Litter collected (kg)	0	10	20
Amount to pay (£)	0	2.5	5

- c) Principal 0.4 Estate manager 0.25 Principal's line is steeper.
 d) Principal $A = 0.4L$ Estate manager $A = 0.25L$
 e) Principal £5.20 Estate manager £3.25

- 3 a) unleaded C lead replacement petrol A diesel B
 b) unleaded $P = 0.809A$ lead replacement petrol $P = 0.869A$ diesel $P = 0.819A$
 where P is the price in £ and A the amount in litres
 c) unleaded £32.36 lead replacement petrol £34.76 diesel £32.76

4 a) and c)

- 5 b) Yes – straight line passing through O.
 c) $h = 2n$ where h is the height in mm and n the number of coins
 d) 36 mm or 3.6 cm
 e) 130 mm or 13 cm
 f) different (steeper) gradient because of different thickness

6

Shape	Feature	Length in Original Shape (x)	Length in Enlargement (y)
Square	Side	1.2 cm	3.6 cm
Circle	Diameter	1.5 cm	4.5 cm
Triangle	Base	1.8 cm	5.4 cm
Triangle	Height	1.3 cm	3.9 cm

- b) 3 (or 1:3)
 d) Gradient = 3, Equation $y = 3x$



Experiments

These could be used to generate evidence for the students' Coursework Portfolio. Some of these could be done in class. For others it would be necessary for students to collect data outside class.

N.B. Wherever possible experiments which are relevant to students' other studies or interests should be used instead.



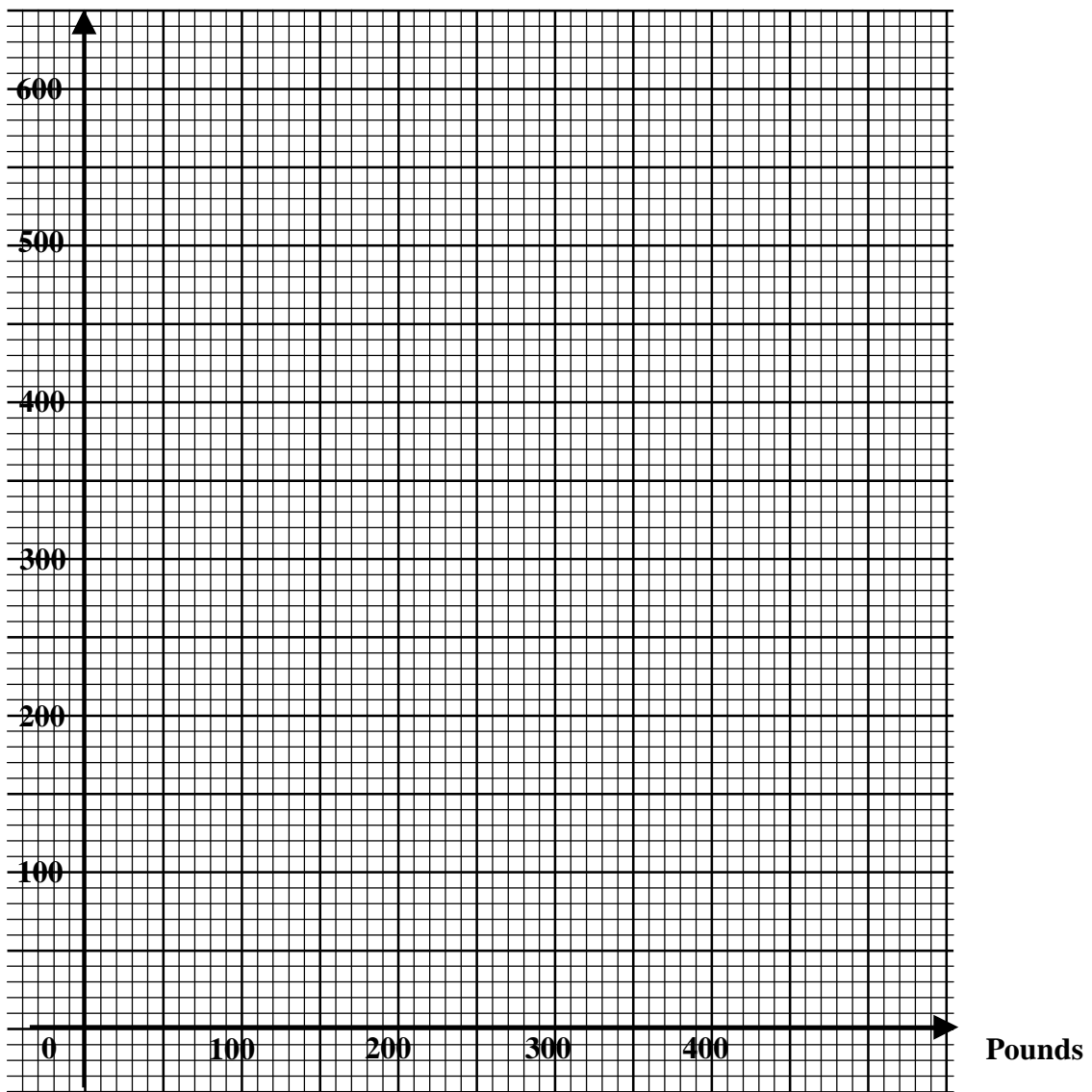
Foreign Currency



Pounds					
Dollars					

Exchanging pounds for dollars

Dollars

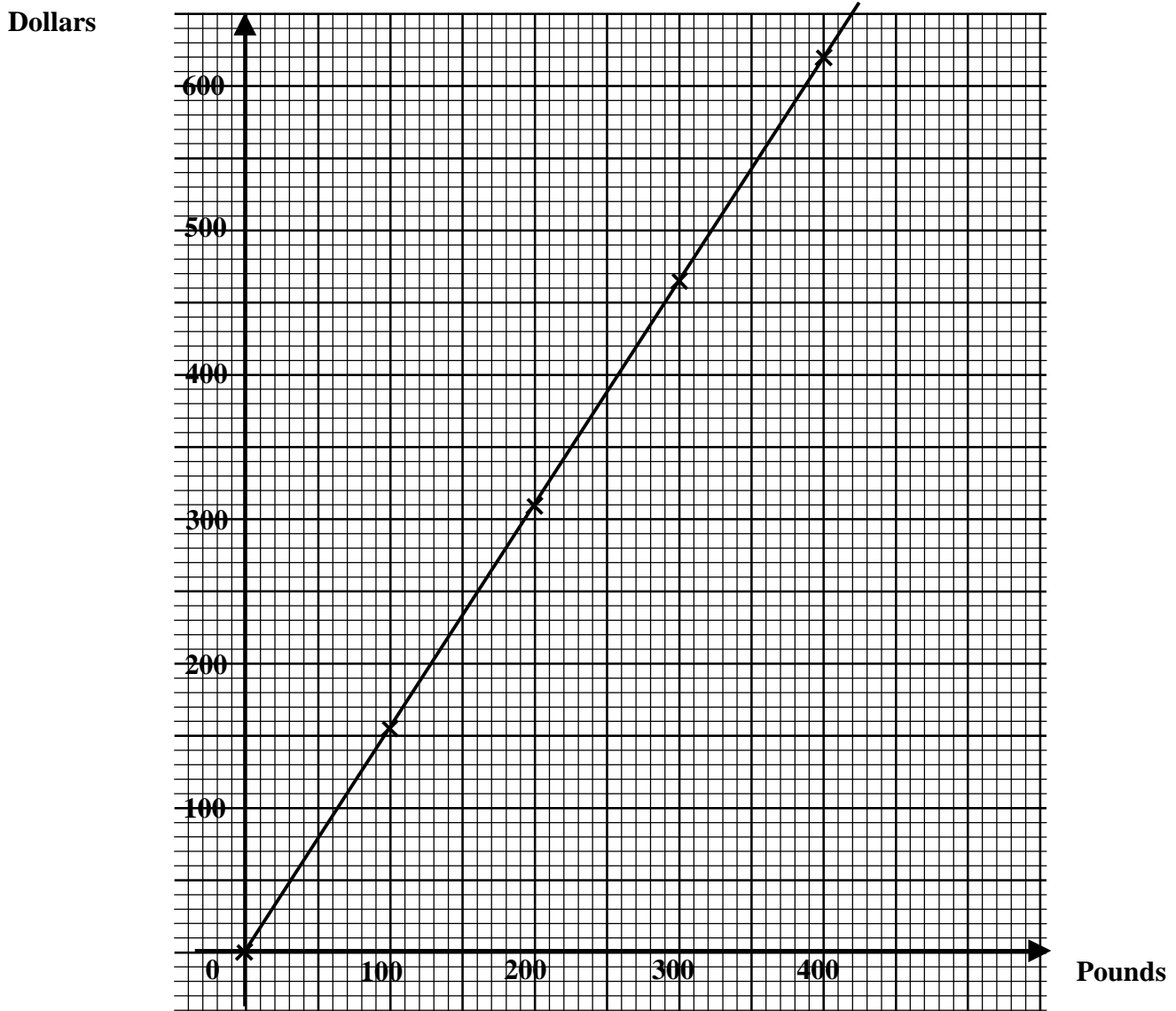


Foreign Currency



Pounds	0	100	200	300	400
Dollars	0	155	310	465	620

Exchanging pounds for dollars



Foreign Currency



£1 = \$1.55

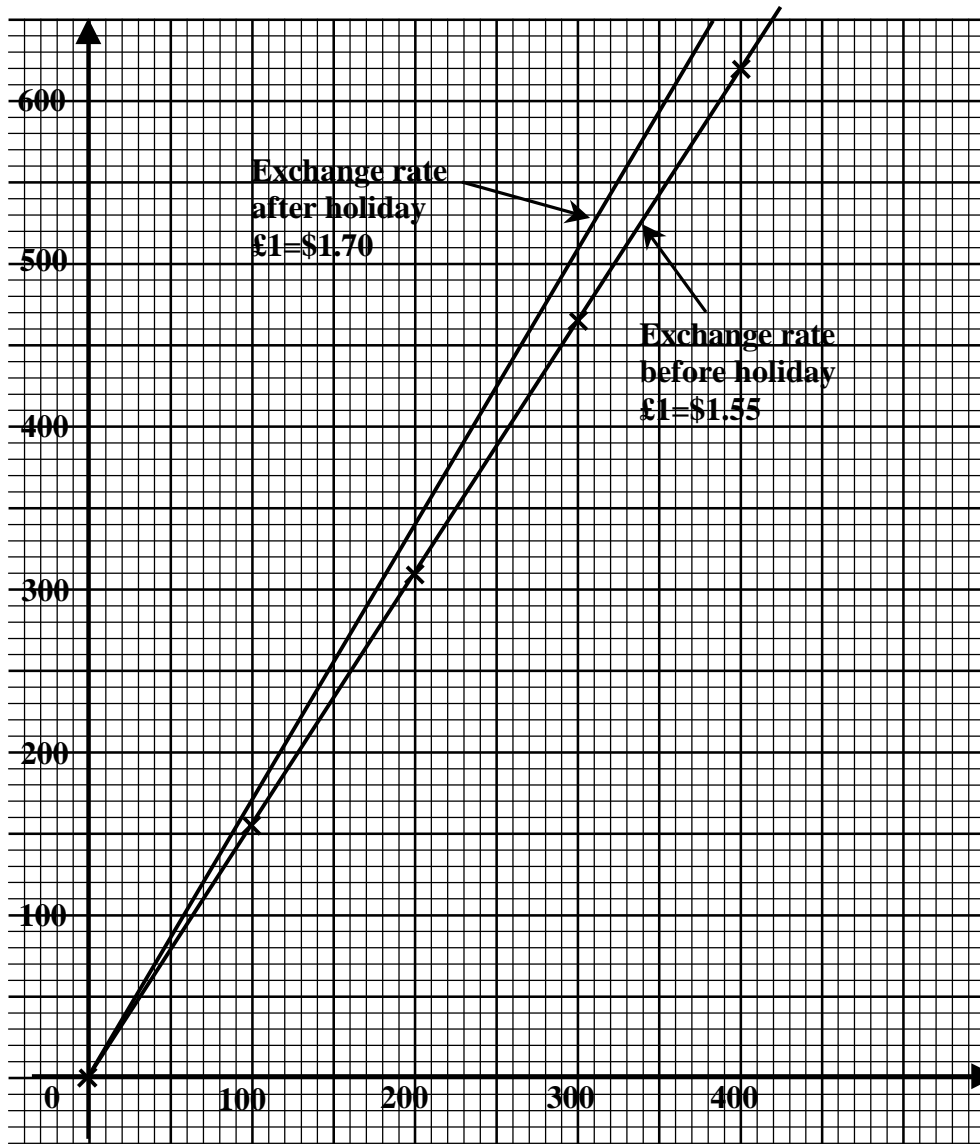
Pounds	0	100	200	300	400
Dollars	0	155	310	465	620

£1 = \$1.70

Pounds	0	100	200
Dollars	0	170	340

Exchanging pounds for dollars

Dollars



Pounds

