AS: Use of Mathematics Work Scheme

AS: Use of Mathematics comprises three equally weighted assessment units:

1. ***FSMQ Working with Algebraic and Graphical Techniques*** (compulsory)
2. either ***FSMQ Using and Applying Statistics*** (optional)
   or ***FSMQ Modelling with Calculus*** (optional)
3. ***Applying Mathematics*** (compulsory)

Each of these units requires 60 guided learning hours giving a total of 180 hours. There are many ways in which a course for AS: Use of Mathematics could be organised.
You may prefer to teach the optional unit in parallel to the compulsory units, after the compulsory units or at some other point in the course. Some teachers use the first term of a one year course to cover the bulk of the compulsory units, then concentrate on the optional unit during the spring term, before revising and completing the compulsory units before the examinations in the summer term.

The work scheme below combines the two compulsory units *Working with Algebraic and Graphical Techniques (WAG)* and *Applying Mathematics (AM)*. This work scheme shows the topics and methods to be covered and suggests time allocations (with a total of 120 hours). Separate work schemes are available for each of the optional units. For the full AS level you will need to cover the work described below and also plan another 60 hours into your course for the optional unit (statistics or calculus).

Although the compulsory topics are listed separately in this work scheme, it would often be beneficial to use a variety of skills within the same piece of work. Some techniques should be introduced as soon as possible and used throughout the course. These include:

- using a calculator effectively and efficiently, recording the working as well as the result and deciding on an appropriate degree of accuracy
- doing calculations without a calculator using written methods and mental techniques
- showing all working by writing clear and unambiguous mathematical statements, including the correct use of brackets
- using notation correctly, including therefore, $\therefore$, equals, =, approximately equals, $\approx$, inequalities, <, >, $\leq$, $\geq$ and implies, $\Rightarrow$
- graph plotting by hand and using either computer software or a graphical calculator
- checking calculations using estimation, inverse operations and different methods and questioning whether solutions are reasonable/valid.

Throughout the course the emphasis should be on the use of algebraic functions to model ***real situations***. Students need to appreciate the main stages in developing a model, understanding that simplifying assumptions are often necessary but may limit the usefulness of solutions. They should interpret the main features of models and consider the validity of any models used. They should understand that a general mathematical model can be used to solve a variety of related problems and use models to predict unknown values.

Topic Area	Content	Nuffield Resource	Coursework Portfolio Requirements
Linear Functions (4 hours)	Revise the main features of graphs of direct proportional ($y = mx$) and linear ($y = mx + c$) functions. Fit such functions to real data using gradients and intercepts. Use error bounds to consider a range of possible functions to model data.	**Interactive Graphs** (WAG starter) Uses interactive spreadsheet graphs to introduce the shape and main features of **proportional**, **linear**, quadratic and power graphs. (Can be split into 3 separate parts.)	**Notes** ***Working with Algebraic and Graphical Techniques*** requires a Coursework Portfolio. The required content of this portfolio is listed on the following pages. ***Applying Mathematics*** does not require a Coursework Portfolio.
Simultaneous Linear Equations and Inequalities (6 hours)	Use algebraic and graphical methods to solve real problems involving linear simultaneous equations and inequalities (on graphs using dashed lines when boundaries are not included, full lines when boundaries are included and shading to indicate regions not included). Use substitution of numerical values in equations and inequalities to verify that solutions are valid.		

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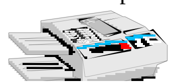

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Quadratic Functions (6 hours)	Draw graphs of quadratic functions of the form: • $y = ax^2 + bx + c$ • $y = (rx - s)(x - t)$ • $y = m(x + n)^2 + p$ relating the shape, orientation and position of the graph to the constants, relating zeros of the function f(x) to roots of the equation f(x) = 0 and developing an appreciation of the symmetry of graphs of quadratic functions. Fit quadratic functions to real data. Rearrange any quadratic function into the forms $y = ax^2 + bx + c$ and $y = a(x + b)^2 + c$ Find maximum and minimum points of quadratics by completing the square.	**Interactive Graphs** (WAG starter) Uses interactive spreadsheet graphs to introduce the shape and main features of proportional, linear, **quadratic** and power graphs. (Can be split into 3 separate parts.) **Test Run** (WAG skills activity) Students interpret a speed-time graph and fit both linear and quadratic models. The performance data is also given in an Excel spreadsheet for comparison with models.	The ***Working with Algebraic and Graphical Techniques*** Coursework Portfolio must contain at least two reports. When taken together these reports should include the requirements listed on the following pages. The way in which these requirements are split between the investigations will depend to a large degree on the contexts involved. Ideally students should investigate real situations from their other areas of study, work or interests and they should use their own data wherever possible. In order to begin portfolio work as soon as possible, teachers will need to plan carefully the order in which they cover topics. The order suggested here may not be the most suitable for some groups and it may well be more appropriate to cover parts of each topic area at separate times, rather than as a block as listed here.
Methods of Solving Equations (applied in real situations wherever possible) (10 hours)	Solve quadratic equations by: • factorising • completing the square • using the formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Use a graphic calculator to solve quadratic and other polynomial equations and simultaneous equations. • Find values of x where $y = \mathrm{f}(x)$ crosses the x axis to solve $\mathrm{f}(x) = 0$. Appreciate that when $\mathrm{f}(x)$ is continuous and $\mathrm{f}(a)$ is of a different sign to $\mathrm{f}(b)$ there is at least one solution of $\mathrm{f}(x) = 0$ between a and b. • Find points of intersection of $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$ to solve $\mathrm{f}(x) = \mathrm{g}(x)$ and develop a graphical understanding of when systems of equations have one or more solutions, no unique solution or no solution. Use algebra to solve simultaneous equations where one is linear and the other quadratic. Understand that in general a system of n equations is needed to find n unknowns. Compare algebraic, graphical and numerical methods of solving equations to develop an appreciation of when a method is appropriate, inappropriate or possibly unsound.		

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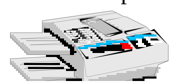
Gradients of Curves, Maxima and Minima (6 hours)	Calculate and understand gradient at a point on a graph using tangents drawn by hand (and also using zoom and trace facilities on a graphic calculator or computer if possible). Use and understand the correct units for rates of change. Interpret and understand gradients in terms of their physical significance. Identify trends of changing gradients and their significance both for known functions and curves drawn to fit data.	**Tin Can** (WAG skills activity) Students design a tin can, using algebraic and graphical techniques. Optional use of the internet.	1 Reports of ***at least two investigations*** including use of: a) different functions as models b) key features of graphs c) algebraic techniques. These should show working in full (especially when calculators have been used) and evidence of estimation and checking to ensure accuracy. In totality the reports should include: a) use of 2 different types of functions to model the same data set, either • *one function for the full data set and another for part of it*, or • *two different functions for different sections of the data* where • at least 1 set of data is plotted using a graphic calculator or computer • the effectiveness of each model is considered • predictions are made for cases where there is no data • explanations are given of how the functions are related to basic functions of their type • errors or inaccuracies in the data are considered and the way in which these may affect the functions used.
Power Functions and **Inverse Functions** (8 hours)	Draw graphs of functions of powers of x including $y = kx^n$ where n is a positive integer, $y = kx^{-1} = \frac{k}{x}$, $y = kx^{-2} = \frac{k}{x^2}$ and $y = kx^{\frac{1}{2}} = k\sqrt{x}$ Learn the general shape and position of such functions and investigate their symmetries. Develop an understanding of the nature of discontinuities (including the occurrence of horizontal and vertical asymptotes). Fit power functions to real data. Find the graph of an inverse function using reflection in the line $y = x$. Solve polynomial equations of the form $ax^n = b$.	**Interactive Graphs** (WAG starter) Uses interactive spreadsheet graphs to introduce the shape and main features of proportional, linear, quadratic and **power** graphs. (Can be split into 3 separate parts.)	



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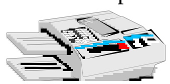
Growth and Decay (6 hours)	Draw graphs of exponential functions of the form $y = ka^{mx}$ and $y = ke^{mx}$ (*m* positive or negative) and understand ideas of growth and decay. Fit exponential models to real data. Recognise how a general mathematical model enables the solution of a variety of problems (eg the use of $a = B \times c^t$ to model radioactive decay where the values of *B* and *c* depend on the substance).	**The Ozone Hole** (WAG skills activity) Data concerning depletion of ozone levels and the increase in the area of the Antarctic ozone hole over the last twenty years. Students investigate possible linear, quadratic and exponential models. Optional use of spreadsheet.	b) use of key features of graphs including: (i) intercepts with axes (ii) gradients (iii) changes and trends in gradients (iv) local maximum and minimum points for functions that model real situations in order to solve problems and explain how the function relates to the real situation. c) use of algebraic techniques to solve problems for: (i) a polynomial model (ii) one other model which may be trigonometric, exponential or logarithmic. (8 hours i.e. 4 hours for each investigation)
Logarithmic Functions (6 hours)	Draw graphs of natural logarithmic functions of the form $y = a\ln(bx)$ and understand the logarithmic function as the inverse of the exponential function. Solve exponential equations of the form $A\exp(mx + c) = k$ Learn and use the laws of logarithms: $\log(ab) = \log a + \log b$, $\log\left(\frac{a}{b}\right) = \log a - \log b$ and $\log(a^n) = n\log a$ to convert equations involving powers to logarithmic form and solve them (using both base 10 and natural logarithms).		
Simulations (required for Applying Mathematics only) (8 hours)	Random events, probability and discrete probability distributions. Use tables and graphic calculators to find random numbers (being aware that graphic calculators generate pseudo-random numbers). Use random numbers to simulate discrete random events. Interpret simulation models being aware of the limitations due to simplifying assumptions and simulations of small numbers of occurrences.	**Queues** (AM starter) Use of random numbers to simulate the queue that builds up in a newsagent's shop.	



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Transformations of Graphs (8 hours)	Use: • translation of $y = \mathrm{f}(x)$ parallel to the *y* axis to give $y = \mathrm{f}(x) + a$ • translation of $y = \mathrm{f}(x)$ parallel to the *x* axis to give $y = \mathrm{f}(x + a)$ • stretch of $y = \mathrm{f}(x)$ parallel to the *y* axis to give $y = a\mathrm{f}(x)$ • stretch of $y = \mathrm{f}(x)$ parallel to the *x* axis to give $y = \mathrm{f}(ax)$ Include a study of the nature of discontinuities of functions of the form $\mathrm{f}(x) = \frac{k}{x}$ and $\mathrm{g}(x) = \frac{k}{x - a}$ and limiting values of functions of the form $\mathrm{P}(t) = Ae^{kt}$ and $\mathrm{g}(x) = K - Ae^{kx}$. Consider the nature of horizontal asymptotes and discuss the way vertical asymptotes may be displayed incorrectly on graphic calculators. Use geometric transformations to assist in fitting functions to real data.	**Water Flow** (WAG assignment) Includes data about the velocity of water as it flows along an open channel and sample examination question. Data could also be used to give practice for portfolio requirements or form the basis for an assignment. **Sea Defence Wall** (WAG assignment) Two versions of an assignment in which students find functions to model the outline of a sea defence wall. The first version encourages students to work independently, the second is more structured for less able students.	(continued on page 6)
Trigonometric Functions (8 hours)	Draw graphs of • $y = A\sin(mx + c)$ • $y = A\cos(mx + c)$ Learn the general shape and position of trigonometric functions and use the terms amplitude, frequency, wavelength, period and phase shift correctly. Fit trigonometric functions to real data and use the symmetry of trigonometric graphs to solve problems. Solve trigonometric equations of the form $A\sin(mx + c) = k$ and $A\cos(mx + c) = k$	**Coughs and Sneezes** (WAG assignment) Includes data about the way in which an outbreak of the common cold spreads. Students are asked to model the data using trigonometric and polynomial functions.	

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Linearising Data (8 hours)	Determine parameters of non-linear laws (in real contexts) by plotting appropriate linear graphs, for example: • $y = ax^2 + b$ by plotting y against x^2 • $y = \frac{a}{x} + b$ by plotting y against $\frac{1}{x}$ • $y = ax^3 + b$ by plotting y against x^3 • $y = a\sin(x) + b$ by plotting y against $\sin(x)$ • $y = ax^b$ and $y = a^x$ using base 10 or natural logarithms	**Log Graphs** (WAG starter) Examples (involving earthquakes and planetary motion) that can be used to introduce log graphs. Ideas of experiments and other situations that can be used for portfolio work. **Smoke Strata** (WAG assignment) Includes data about the height of smoke layers due to a fire in a tall building and sample examination question. Data could also be used to give practice for portfolio requirements or form the basis for an assignment.	2 Report of a piece of work in which a function is fitted to non-linear data using logarithmic or other techniques to determine parameters by plotting a linear function. Include clear explanation of method, correct algebraic notation and clearly labelled, accurate graphs. (4 hours) (Total for Coursework Portfolio = 12 hours)
Recurrence Relations (required for Applying Mathematics only) (6 hours)	Investigate discrete models using recurrence relations in applications such as population growth (including birth and death rates) and investment. Include the use of subscript notation, finding a sequence of values using an initial value, x_0 and a relation between x_{n+1} and the previous term x_n. Plot a graph of x_n against n to illustrate the results. Understand the difference between discrete and continuous models and between recurrence relations such as $a_{n+1} = ka_n + b$ and closed forms such as $u_n = a + kn$.		
Making Sense of Mathematics (required particularly for Applying Mathematics) (6 hours)	Read and understand mathematical work done by somebody else. Explain steps in mathematical working, developing sub-steps where necessary. Relate mathematics in new situations to mathematics in familiar situations. Develop strategies such as considering boundary conditions, extreme values and simple values to help make sense of mathematics. Develop alternative representations (algebraic, graphical or numerical) to help explain the mathematics.	**Mortality** (AM comprehension) Article involving the historical development of the theory of mortality used by insurance companies and questions based on the article.	
Revision (12 hours)