

Modelling with Calculus Work Scheme

This FSMQ requires a total of 60 guided learning hours that could be timetabled in a variety of ways eg 2 hours per week for 30 weeks, 4 hours per week for 15 weeks, 5 hours per week for 12 weeks. A suggested work scheme showing topics and methods to be covered is given below but the order and time allocations can be varied to suit different groups of students. Note that before starting the course students are expected to:

- be able to use algebraic methods to rearrange and solve linear and quadratic equations
- be familiar with the graphs of basic functions (powers of x , quadratic, trigonometric, exponential and logarithmic functions) and how to transform them using translations and stretches parallel to the x and y axes.

The following techniques should be introduced as soon as possible and used throughout the course:

- using a calculator effectively and efficiently, including the use of memory and function facilities and recording the working as well as the result
- doing calculations without a calculator using written methods and mental techniques
- graph plotting by hand and using either computer software or a graphical calculator (including zoom and trace facilities if possible)
- checking calculations using estimation, inverse operations and different methods.

Topic Area	Content	Nuffield Resource	Coursework Portfolio Requirements
Introduction to calculus (4 hours)	What is calculus? Brief revision of gradients of straight lines and curves including real contexts. Positive, negative and zero gradients and their interpretation. Sketching graphs of gradient functions. Finding and interpreting area under graphs (eg speed/time, acceleration/time) using areas of triangles, rectangles and trapezia.		Note When taken together the reports should include the requirements listed below. The way in which these requirements are split between the investigations will depend to a large degree on the contexts involved. Ideally students should investigate real situations from their other areas of study, work or interests.
Gradient functions (6 hours)	Gradients of chords leading to the numerical approximation: gradient of tangent $\approx \frac{f(a+h) - f(a)}{h}$ where h is small. Use $\frac{f(a+h) - f(a)}{h}$ to generate gradient data and sketch graphs of gradient functions. Use of $\frac{f(x+h) - f(x)}{h}$ to generate a gradient function. Gradient of $y = x^n$ is $\frac{dy}{dx} = nx^{n-1}$. Differentiate polynomials, sums and differences, functions multiplied by a constant using notation $\frac{dy}{dx}$ and $f'(x)$ Include units and interpretation of gradients and rates of change.	Gradients (starter) Powerpoint presentation introducing differentiation and a worksheet and Excel spreadsheet which students can use to calculate gradients from increments.	In order to begin portfolio work as soon as possible, teachers will need to plan carefully the order in which they cover topics. The order suggested here may not be the most suitable for some groups and it may well be more appropriate to cover parts of each topic area at separate times, rather than as a block as listed here.
		Polynomial graphs and gradients (skills activity) 12 sets of cards, each consisting of a polynomial function, its graph, its gradient function and the graph of the gradient function – for students to match.	



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Areas under curves (5 hours)	Use of trapezium rule or mid-ordinate rule (+ Simpson's rule if you wish). Over and under-estimates, improving accuracy by using a smaller interval.	Coastal Erosion A (skills activity) This activity uses the context of coastal erosion to introduce the trapezium rule for estimating the area under a curve. (Could be combined with Coastal Erosion B.)	<p>Reports of <i>at least two investigations</i> that use functions to model the situation and make use of:</p> <ol style="list-style-type: none"> differentiation integration numerical methods differential equations. <p>These should show working in full (especially when calculators have been used) and evidence of estimation and checking to ensure accuracy.</p> <p>In totality the reports should include:</p> <ol style="list-style-type: none"> the use of differentiation to solve problems, including using graphs and expressions of first and second derivatives to help: <ol style="list-style-type: none"> describe clearly how gradients and changing gradients relate to the situation; find and interpret stationary points; indicate key features (in sketch graphs) and relating them to the real situation.
Integration (6 hours)	Find areas under curves, between $x = a$ and $x = b$ using $\int_a^b f(x) dx$, ($f(x) \geq 0$) Simple integration rules including sums, differences and multiplication by a constant including the use of correct notation and constant of integration. Definite integration	Coastal Erosion B (skills activity) Students use integration to estimate loss of land due to coastal erosion. (Could be combined with Coastal Erosion A.)	
		Area under a graph (starter) Introduces integration using the area under velocity-time graphs. Powerpoint presentation, notes and exercises.	
		Mean Values (skills activity) Students use area formulae and integration to find mean values of speed, share prices and water depth. Supported by Powerpoint presentation.	
Second derivatives (5 hours)	Using gradients to identify key features - maxima, minima and points of inflexion. Finding second derivatives using notation: $\frac{d^2y}{dx^2}$ and $f''(x)$ Interpreting second derivatives and using them to find stationary points: maxima, minima and points of inflexion. Include fact that zero values of second derivatives can occur at maxima and minima as well as points of inflexion.	Stationary Points (skills activity) Powerpoint presentation, examples and practice questions on sketching graphs.	
		Maxima and Minima (skills activity) Powerpoint presentation and practice questions using differentiation to solve maxima and minima problems.	
		Containers (assignment) Assignment in which students use differentiation to minimise surface area.	
More rules of differentiation (6 hours)	Differentiating <ul style="list-style-type: none"> trigonometric functions (radians needed) exponential functions products functions of functions any other functions which are appropriate. 	Exponential Rates of Change (starter) Introduction to differentiation of exponential functions. Students draw tangents to curves, then investigate gradient functions using a spreadsheet.	



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More integration (4 hours)	Integration of <ul style="list-style-type: none"> trigonometric functions $A \sin(mx + c)$, $A \cos(mx + c)$ exponential functions ke^{mx} (m positive or negative) any other appropriate functions. Applications of integration.	That's a Lot of Rock! (assignment) Students fit a curve to the cross section of a tunnel, then use integration and numerical methods to estimate the volume of rock removed and the time taken.	b the use of integration to find the area under a curve in <i>two</i> different situations where the area has physical significance (using correct notation and showing each stage of the working clearly) c an investigation of the effectiveness of using numerical methods of differentiation and integration compared with using analytic methods (showing clearly all stages of the working and giving a reasoned discussion of the effectiveness of the numerical methods used). d the use of a simple differential equation which is solved to find a function that models the situation under investigation (including a clear and accurate graph to illustrate the work and solution). (Total time allocation for investigations 12 hours)
Differential equations (6 hours)	Sketching direction-field diagrams: $\frac{dy}{dx} = f(x)$, $\frac{dy}{dx} = f(y)$ where $f(y) = k$ and $f(y) = ay^n$ Using integration to find families of solutions of simple differential equations. Particular solutions of simple differential equations using boundary conditions.	Drug Clearance (skills activity) Data Sheet shows how drug clearance after taking a painkiller can be modelled by exponential decay. . Students investigate further (including the clearance of caffeine after a variety of drinks). What's it worth? (skills activity) Investigate a variety of suggested models for the depreciation of a car. Involves solving differential equations, drawing sketch graphs and comparison with real data.	
Revision (6 hours)			

