Mathematical Principles for Personal Finance Work Scheme (Pilot 2008-9)

Before starting this course students should have acquired the skills and knowledge associated with the Number & Algebra assessment objective of GCSE Mathematics, or with an Intermediate (level 2) FSMQ Finance course, or equivalent.

A suggested work scheme showing topics and methods to be covered is given below. This recommends a total of 60 guided learning hours (eg 2 hours per week for 30 weeks, 4 hours per week for 15 weeks, 5 hours per week for 12 weeks). The order in which the topics are covered and the time allocated to them can be varied to suit different groups of students.

Topic Area	Content	Nuffield Resources
Percentages & Personal Taxation (3 hours)	Calculate % change = $\frac{\text{current value} - \text{previous value}}{\text{previous value}} \times 100$ Use multipliers to combine % changes (including repeated changes) and to reverse % changes - including VAT. Carry out complex calculations involving multiple rates – including income tax, national insurance, capital gains tax	**Work with %** Discussion and worksheets supported by PowerPoint presentation – check what methods students use for some real-life % problems and introduce/revise the use of multipliers. **Income Tax** Notes, examples and exercises based on rates for the current tax year. **National Insurance** Notes, examples and exercise based on rates for the current tax year.
Saving (4 hours)	Class discussion - Would it be better to receive a sum of money (eg £5000) now or at a later date (eg 20 years from now)? Discuss the key idea of present value (PV) and future value (FV). Collect and discuss information about interest rates from banks, building societies and other sources. (Include information given in the form of tables and diagrams.) Calculate the future value of a present sum (using ideas of compound interest) leading to the use of recurrence relations eg $P_{n+1} = P_n(1+r)$. Include the use of a calculator and spreadsheet (if possible). AER: Calculate the annual effective interest rate, *r*, given a nominal interest rate, *i* $r = \left(1 + \frac{i}{n}\right)^n - 1$ where *n* is the number of compounding periods per year.	**Savings Growth** Leaflet giving interest rates on three savings accounts and worksheet based on the leaflet. This introduces the idea of a recurrence relation and includes the use of a graphic calculator, Excel spreadsheet and formulae to find future values and the AER.

<table>
<tr><th>Topic Area</th><th>Content</th><th>Nuffield Resources</th></tr>
<tr><td rowspan="3">Indices
(5 hours)</td><td rowspan="3">Use data relevant to students' other studies or interests to introduce the idea of an index as a ratio that describes the relative change in a variable (e.g. price) compared to a certain base period (e.g. one year). Include examples from the National Statistics website at www.statistics.gov.uk (eg indices for prices, earnings, building costs, manufacturing output, motor vehicle production, retail sales volumes etc.)
Include % change = $\frac{\text{current index} - \text{previous index}}{\text{previous index}} \times 100$
for areas of finance such as the FTSE 100 share index.

Research and compare the Retail Price Index (RPI) and Consumer Price Index (CPI).
Include calculation from the Laspeyres index formula (weighted by quantities in the base period):
$I_L = \frac{\sum P_{it} Q_{i0}}{\sum P_{i0} Q_{i0}} \times 100$ where P_{it} is the price of commodity i at time t,
and Q_{it} is the quantity of commodity i at time t
0 represents the base period so for example Q_{i0} represents the quantity of commodity i at the base period ($t = 0$)</td><td>Average Earnings Index
Learners use average earning index numbers to compare the way in which earnings in different sectors have changed over different time periods.</td></tr>
<tr><td>Inflation Indices
Activity in which students find information about RPI and CPI and answer questions – will provide a set of notes.</td></tr>
<tr><td>Laspeyres Index
This activity uses a simple coffee shop example to introduce the Laspeyres index formula, gives learners practice in real contexts then asks them to calculate a Laspeyres index for their own regular spending pattern. (Includes optional use of the Personal Inflation Calculator on the National Statistics website).</td></tr>
<tr><td>Data over time
(4 hours)</td><td>Inspect tables & graphs considering changes over time & interpreting trends (eg in stocks & shares, interest rates, exchange rates). Represent data graphically and identify linear trends (using gradient & intercept and algebraic substitution).
Calculate average changes. Use moving averages to smooth short term fluctuations and interpret situations which may include seasonal and cyclical variations.
Include moving average at interval m with n data points $\overline{x_m} = \frac{p_m + p_{m-1} + p_{m-2} + \ldots . p_{m-(n-1)}}{n}$
and calculate successive values using $\overline{x_m} = x_m - \frac{p_{m-(n-1)}}{n} + \frac{p_{m+1}}{n}$
Addressing the problem of lag using weighting and calculate the linear weighted moving average $\overline{x_m} = \frac{np_m + (n-1)p_{m-1} + (n-2)p_{m-2} + \ldots . p_{m-(n-1)}}{n + (n-1) + (n-1) + \ldots + 2 + 1}$ where the denominator is the triangular number with sum $\frac{n(n+1)}{2}$</td><td>.</td></tr>
</table>

Topic Area	Content	Nuffield Resources
Borrowing (3 hours)	APR (annual percentage rate) Use the simplified version formula for APR for a single loan repaid in full after a fixed period: $C = \frac{A}{(1+i)^n}$ Overall cost, where n = number of years between start of loan and its repayment.	APR – Annual Percentage Rate Introduces the formula for APR that applies when a single loan is repaid in full after a fixed period and gives learners practice in using it.
Financial diagrams (3 hours)	Make sense of a range of information presented in tables and diagrams relating to finance. Include tabulated information about savings, loans, credit card accounts, shares, ISAs (etc.) and information in the form of graphs and charts.	
Weighted Averages (4 hours)	Calculate contributions made by individual items to indices, e.g. calculate contributions made by the prices of commodities in different shops and regions to a consumer price index. For example, if a commodity costs £5 in shop A and £6 in shop B and 40% (0.4) of customers buy the commodity from shop A and 60% (0.6) from shop B, the effective cost of the commodity used in calculating an index is 0.4 × £5 + 0.6 × £6 = £5.60 Calculate a composite index by combining indices using weighting (e.g. in calculating a price index the index of each commodity multiplied by its weighting is totalled and this sum is divided by the sum of all the commodities weights).	
Further Borrowing (8 hours)	Use more complex recurrence relations eg for credit cards, mortgages. Find APR (annual percentage rate) for a loan repaid in a small number of instalments (eg 2, 3 or 4). using loan $C = \frac{A_1}{1+i} + \frac{A_2}{(1+i)^2} + \frac{A_3}{(1+i)^3} + \frac{A_4}{(1+i)^4}$ by: • substituting values into the above equation for confirmation, • solving the above equation for *i* using the interval bisection method.	**Credit Cards** Learners use recurrence relations to work out how long it takes to pay off credit card debts. Includes the use of both a graphic calculator and spreadsheet. **APR in more difficult cases** This shows learners how to find and check APR values in cases where a loan is paid back in more than one instalment. Includes plenty of practice in using the interval bisection method and accompanied by a spreadsheet that is set up to use this method to calculate APRs.

Topic Area	Content	Nuffield Resources
Further Indices (8 hours)	Use the **Paasche** index formula (weighted by quantities in the calculation period): $I_P = \frac{\sum P_{it} Q_{it}}{\sum P_{i0} Q_{it}} \times 100$ where P_{it} is the price of commodity i at time t, and Q_{it} is the quantity of commodity i at time t Research and compare fixed base and chain linked indices: to develop an understanding that in a chain index comparisons are always made between subsequent points and therefore take account of changes between the start and end points **fixed base index** $t = 0$ $t = 1$ $t = 2$ $t = 3$ **chain index** Use the Fischer index formula $I_F = \sqrt{I_L \times I_P}$ (geometric mean of Laspeyres & Paasche indices) Compare Laspeyres, Paasche & Fischer indices.	
More tables & diagrams (6 hours)	Use information given in more complex tables relating to personal finance. Discuss and make sense of a range of less familiar charts eg hi-low charts, candlestick charts, Kagi charts.	
Continuous compounding (4 hours)	Introduce and use the idea that continuous compounding leads to exponential functions ie $P = P_0\left(1 + \frac{r}{n}\right)^{nt}$ is the amount after t years for an initial investment of P_0 when the interest is compounded n times per year, and $n \to \infty$ giving $P = P_0 e^{rt}$	
Revision (8 hours)	Discuss & work through revision questions **Discuss Data Sheet and make up and work questions based on it.**	**Financial Calculations** A set of worked examples to help with revision.