

Algebra and Graphs Work Scheme (Pilot 2008-9)

Before starting this Intermediate (Level 2) FSMQ students should be able to:

- calculate with numbers (including large and very small numbers) by carrying out operations in the correct order, working both by hand and with a calculator
- calculate with fractions and decimals (including expressing one quantity as a fraction of another and converting between decimals and fractions)
- calculate with percentages
- round values to the nearest whole number, 10, 100, 1000, 1/10 (0.1), 1/100 (0.01) etc.
- substitute into formulae expressed in words and symbols

A suggested work scheme showing topic areas and methods to be covered is given below. This recommends a total of 60 guided learning hours that could be timetabled in a variety of ways eg 2 hours per week for 30 weeks, 4 hours per week for 15 weeks, 5 hours per week for 12 weeks. There is plenty of scope for varying the order and time allocation as many of the mathematical techniques can be introduced/revised in several different contexts. Although the topics are listed separately, it would often be beneficial to use a variety of skills within the same piece of work. Some techniques should be introduced as soon as possible and used throughout the course. These include:

- using a calculator effectively and efficiently (preferably a graphic calculator), including the use of standard form for large and small values
- doing calculations without a calculator using written methods and mental techniques
- recording and presenting data in tables using an appropriate degree of accuracy and correct units (grouping data where appropriate)
- graph plotting by hand and using either computer software or a graphic calculator
- checking calculations using estimation, inverse operations and alternative methods.

Topic Area	Content	Nuffield Resource
Plot Graphs of Real Data (5 hours)	Plot accurate graphs of data pairs by hand and using either a graphic calculator or function plotting software, ensuring that the graphs are correctly scaled and labelled. Average data if necessary (mean, mode, median). Use coordinates in all 4 quadrants where appropriate. Fit lines by eye (both straight and curved, as appropriate). Find the intercepts of linear and non-linear graphs with axes and where appropriate understand their physical significance. Use trace facilities with graphic calculators or function plotting software where possible. Identify errors in data by inspection of the data set and by graphical means. Recognise that measurements expressed to a given unit can have a maximum error of half a unit; using notation such as 300 ± 50 to express errors.	Road Test (starter) Use data from a road test on a sports car for practice in drawing and interpreting graphs. Optional use of spreadsheet.
Linear Graphs (3 hours)	Calculate in appropriate units and understand the physical significance of the gradients of linear graphs. Consider the intercepts of linear graphs with axes and where appropriate understand their physical significance.	Hire a Coach (starter) Introduces the concepts of gradient and intercept for linear graphs using Excel.
Standard Form (2 hour)	Convert between standard form and ordinary numbers. Use a calculator to perform calculations with numbers expressed in standard form.	Large and Small (starter) Powerpoint presentation and examples from real contexts to introduce, practise or revise standard form.



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Graphs of Functions (6 hours)	<p>Use functions to find data pairs: $y = mx + c$, $y = kx^2 + c$,</p> $y = ax^2 + bx + c, \quad y = ax^3 + bx^2 + cx + d, \quad y = \frac{k}{x},$ <p>and functions with other variables eg $P = \frac{k}{V}$.</p> <p>Use function notation eg $f(x) = kx^2 + c$</p> <p>Look for patterns that data fitting proportional ($y = mx$), linear ($y = mx + c$) and quadratic models of the form $y = kx^2$ have.</p> <p>Plot graphs of functions, using co-ordinates in all four quadrants. Consider the main features of direct and indirect proportional, linear and quadratic models and their differences. Predict the shapes of graphs of direct and inverse proportion, linear and quadratic functions from an algebraic statement.</p>	Linear Graphs (starter) Powerpoint presentation and activity to introduce linear graphs.
		Graphs of Functions in Excel (starter) This activity shows students how to draw graphs of algebraic functions in Excel.
		Spreadsheet Graphs (skills activity) Interactive spreadsheet graphs introducing the shape and main features of proportional, linear, inverse proportional and quadratic graphs.
		Linear Relationships (skills activity) Example and exercise involving proportionality and other linear relationships in scientific contexts.
		Quadratic Graphs (skills activity) Powerpoint presentation, notes and exercise on drawing quadratic graphs and using them to solve equations.
Substitution into Formulae including Conversion of Units (3 hours)	<p>Convert within and between metric and imperial systems, including inches, feet, yards, miles using conversion factors and the use of formulae eg $L = 3.28l$ for converting metres to feet.</p> <p>Substitute data into other formulae to find secondary data including formulae with multiples and fractions of linear terms, powers (positive and negative integers and fractions) and brackets.</p>	Formulae (skills activity) Powerpoint presentation, notes and exercise including a range of formulae involving areas and volumes, interest calculations, temperature conversion and equations of motion.
Re-arrange Algebraic Expressions (3 hours)	<p>Rearrange algebraic expressions by:</p> <p>(i) collecting like terms eg sums and differences of terms such as, na, nab, na^2</p> <p>(ii) expanding brackets eg $2(x + y)$, $a(2a + 3b)$, $\frac{1}{2}(4x^2 + 6x)$</p> <p>(iii) extracting common factors eg expressions such as $(4x + 6x^2)$, $(16y - 8xy)$</p>	



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Areas under Graphs (4 hours)	Calculate estimates of areas under graphs and understand their physical significance (if any) using the formula for the area of a trapezium (and triangle if necessary).	Area under a graph (starter) Powerpoint to introduce area under a velocity-time graph accompanied by examples for students to try. Optional use of spreadsheet/graphic calculator.
Rearrange Formulae (4 hours)	Include examples such as: <ul style="list-style-type: none"> ▪ to give u if $v = u + 10t$ ▪ to give I if $P = 1000 I^2$ or $P = I^2 R$ ▪ to give P if $A = P + \frac{PRT}{100}$ 	Rearrange Formulae (skills activity) A wide range of formulae from real contexts (areas, volumes, interest calculations, temperature conversion, equations of motion etc) for students to rearrange. Includes cards that can be used to help students with the most difficult cases.
Line and Curve Fitting (6 hours)	Recognise the main features of the data and graphs of: <ul style="list-style-type: none"> • direct proportional ($y = mx$) and linear ($y = mx + c$) models and be aware of their differences • quadratic models of the form $y = kx^2$ Recognise the graphs of inverse proportional models Use the gradient and intercept of a straight line fitted to data to find an algebraic statement for it. Understand when it is appropriate or not to use a particular function to model data by consideration of intercepts, long term behaviour etc. in real world terms. Find a function to fit data using substitution of values into a given expression for the model $\left(y = mx + c, y = kx^2 + c, y = \frac{k}{x} \right)$	Match linear functions and graphs (skills activity) 12 sets of cards, each containing a linear graph, its equation and the real situation it represents – for students to match.
		Road Test Data (skills activity) Includes data from a road test on a sports car. Worksheet giving practice in fitting linear and quadratic functions. Optional use of spreadsheet.
		Boyle’s Law (assignment) Data sheet giving pressure and volume of a fixed mass of gas, an assignment and sample examination question based on this experimental data.
		Shoot (assignment) Students investigate the distance travelled by an object rolling down an inclined plane using graphical and algebraic techniques. Includes a guidance sheet.
		Experiments (skills activity) List of seven experiments that generate linear and non-linear data. Students find appropriate algebraic models.

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<p>Solve Equations (5 hours)</p>	<p>Using graphs</p> <ul style="list-style-type: none"> ▪ to determine a value of a when you know $f(a)$ ▪ to solve equations involving functions of the form $y = mx + c$, $y = kx^2 + c, y = kx^3, y = \frac{k}{x},$ <p>and other functions useful to students' other work.</p> <p>Using algebra</p> <ul style="list-style-type: none"> • to form and solve exactly equations where the unknown appears in only one term including where the unknown is squared e.g $2x^2 + 14 = 20 \Rightarrow x = \sqrt{3}$ or $-\sqrt{3}$ ▪ to form and solve exactly equations where the unknown appears in two terms, each of the same power eg $4x - 2 = 2x + 8, 3x^2 + 4 = 20 - x^2$ 	<p>.</p>
<p>Solve Simultaneous Equations (5 hours)</p>	<p>Find the approximate solution of linear simultaneous equations by finding the point of intersection of two straight lines.</p> <p>Form and solve pairs of linear simultaneous equations by an algebraic method.</p>	<p>Plumbers' Prices (starter) Introduction to the graphical solution of simultaneous equations using Excel in real contexts. Can be used as a follow-up to Hire a Coach.</p> <p>Circuit Boards (assignment) Students investigate the cost efficiency of two machines using graphical and algebraic techniques. Includes a guidance sheet.</p>
<p>Solve Quadratic Equations (6 hours)</p>	<p>Form and solve quadratic equations by</p> <ul style="list-style-type: none"> • factorising with $a = 1$ • using the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ 	<p>Factor Cards (skills activity) Nearly 100 pairs of cards showing a wide variety of quadratic expressions and their factors. Use in a pairing activity to give students practice in expanding brackets or factorising.</p> <p>Quadratic Formula (skills activity) Two examples introducing the quadratic formula and a set of similar equations to solve.</p> <p>Tunnel (skills activity) Students solve a problem involving a road tunnel by finding solutions of quadratic equations using a graph drawn in Excel and then using the quadratic formula. Includes a range of other problems.</p>
<p>Revision (8 hours)</p>	<p>Revise topics and try past papers. Discuss Data Sheet – make up and try questions based on it.</p>	