Working with Algebraic and Graphical Techniques Work Scheme

This FSMQ requires a total of 60 guided learning hours that could be timetabled in a variety of ways eg 2 hours per week for 30 weeks, 4 hours per week for 15 weeks, 5 hours per week for 12 weeks. A suggested work scheme showing topics and methods to be covered is given below. Although the topics are listed separately, it would often be beneficial to use a variety of skills within the same piece of work. Some terms and techniques should be introduced as soon as possible and used throughout the course. These include:

- using a calculator effectively and efficiently, including the use of memory and function facilities and recording the working as well as the result
- doing calculations without a calculator using written methods and mental techniques
- rearranging algebraic expressions by collecting like terms, expanding brackets and extracting common factors
- graph plotting by hand and using either computer software or a graphical calculator
- checking calculations using estimation, inverse operations and different methods.

Topic Area	Content	Nuffield Resource	Coursework Portfolio Requirements
Linear Functions (3 hours)	Revise the main features of graphs of direct proportional ($y = mx$) and linear ($y = mx + c$) functions. Fit such functions to real data using gradients and intercepts. Understand whether it is appropriate or not to use a particular function to model data by consideration of intercepts, long term behaviour (etc.) in real world terms. Use error bounds to consider a range of possible functions to model data. Solve linear simultaneous equations using graphical and algebraic methods.	**Linear Graphs** (starter) Powerpoint presentation and activity to introduce linear graphs. **Graphs of Functions in Excel** (starter) This activity shows students how to draw graphs of algebraic functions in Excel. **Interactive Graphs** (starter) Uses interactive spreadsheet graphs to introduce the shape and main features of proportional, linear, quadratic and power graphs. **(Can be split into 3 separate parts.)** **Using the CASIO fx-7400G PLUS** (skills activity) Notes on how to use this calculator - includes how to draw the graph of a function, investigate how well a model fits data and how to find a model. **Match linear functions and graphs** (skills activity) 12 sets of cards, each containing a linear graph, its equation and the real situation it represents – for students to match.	**Note** When taken together the reports should include the requirements listed on the following pages. The way in which these requirements are split between the investigations will depend to a large degree on the contexts involved. Ideally students should investigate real situations from their other areas of study, work or interests and they should use their own data wherever possible. In order to begin portfolio work as soon as possible, teachers will need to plan carefully the order in which they cover topics. The order suggested here may not be the most suitable for some groups and it may well be more appropriate to cover parts of each topic area at separate times, rather than as a block as listed here.

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Quadratic Functions (7 hours)	Draw graphs of quadratic functions of the form: • $y = ax^2 + bx + c$ • $y = (rx - s)(x - t)$ • $y = m(x + n)^2 + p$ relating the shape, orientation and position of the graph to the constants and relating zeros of the function f(x) to roots of the equation f(x) = 0. Fit quadratic functions to real data. Revise solving quadratic equations by: • factorising • using the formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Rearrange any quadratic function into the forms $y = ax^2 + bx + c$ and $y = a(x + b)^2 + c$ Find maximum and minimum points of quadratics by completing the square.	**Test Run** (skills activity) Students interpret a speed-time graph and fit both linear and quadratic models. The performance data is also given in an Excel spreadsheet for comparison with models. **Broadband A, B, C** (skills activity) Instructions showing how to use Excel, a graphic calculator & algebra to find a quadratic model for the growth in broadband connections in recent years. Powerpoint presentation shows the algebra version. **Factor Cards** (skills activity) Nearly 100 pairs of cards showing a wide variety of quadratic expressions and their factors. Pairing will give students practice in expanding brackets or factorising. **Water Flow** (assignment) Includes data about the velocity of water as it flows along an open channel and sample examination question. Data could also be used to give practice for portfolio requirements or form the basis for an assignment.	1 Reports of ***at least two investigations*** including use of: a) different functions as models b) key features of graphs c) algebraic techniques. These should show working in full (especially when calculators have been used) and evidence of estimation and checking to ensure accuracy. In totality the reports should include: a) use of 2 different types of functions to model the same data set, either • *one function for the full data set and another for part of it*, or • *two different functions for different sections of the data* where • at least 1 set of data is plotted using a graphic calculator or computer • the effectiveness of each model as a model is considered • predictions are made for cases where there is no data • explanations are given of how the functions are related to basic functions of their type • errors or inaccuracies in the data are considered and the way in which these may affect the functions used.
Gradients of Curves, Maxima and Minima (3 hours)	Calculate and understand gradient at a point on a graph using tangents drawn by hand (and also using zoom and trace facilities on a graphic calculator or computer if possible). Use and understand the correct units for rates of change. Interpret and understand gradients in terms of their physical significance. Identify trends of changing gradients and their significance both for known functions and curves drawn to fit data.	**Tin Can** (skills activity) Students design a tin can, using algebraic and graphical techniques. Optional use of the internet. **Max & Min Problems** (skills activity) Powerpoint presentation and practice questions using a spreadsheet or graphic calculator to solve problems involving maximum and minimum values.	
Power Functions and **Inverse Functions** (3 hours)	Draw graphs of functions of powers of x including $y = kx^n$ where n is a positive integer, $y = kx^{-1} = \frac{k}{x}$, $y = kx^{-2} = \frac{k}{x^2}$ and $y = kx^{\frac{1}{2}} = k\sqrt{x}$ Learn the general shape and position of such functions. Find the graph of an inverse function using reflection in the line $y = x$. Solve polynomial equations of the form $ax^n = b$.	**Interactive Graphs** (starter) See above.	

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Growth and Decay (8 hours)	Draw graphs of exponential functions of the form $y = ka^{mx}$ and $y = ke^{mx}$ (m positive or negative) and understand ideas of growth and decay. Draw graphs of natural logarithmic functions of the form $y = a\ln(bx)$ and understand the logarithmic function as the inverse of the exponential function. Solve exponential equations of the form $A\exp(mx + c) = k$ Learn and use the laws of logarithms • $\log(ab) = \log a + \log b$ • $\log\left(\frac{a}{b}\right) = \log a - \log b$ • $\log(a^n) = n\log a$ to convert equations involving powers to logarithmic form and solve them (using both base 10 and natural logarithms).	**Growth and Decay** (starter) Powerpoint presentation that uses compound interest and radioactive decay to introduce exponential growth and decay. **The Ozone Hole** (skills activity) Data concerning depletion of ozone levels and the increase in the area of the Antarctic ozone hole over the last twenty years. Students investigate possible linear, quadratic and exponential models. Optional use of spreadsheet. **Climate A, B** (skills activity) Students use an Excel spreadsheet/graphic calculator to find polynomial functions to model temperature change and compare with exponential models. **Cup of Coffee** (assignment) Data Sheet gives the amount of caffeine remaining in the bodies of a group of people at intervals of 1 hour after they have drunk a cup of coffee or cola. Students are asked to model the data (exponential and linear functions).	b) use of key features of graphs including: (i) intercepts with axes (ii) gradients (iii) changes and trends in gradients (iv) local maximum and minimum points for functions that model real situations in order to solve problems and explain how the function relates to the real situation. c) use of algebraic techniques to solve problems for: (i) a polynomial model (ii) one other model which may be trigonometric, exponential or logarithmic. (8 hours i.e. 4 hours for each investigation)
Transformations of Graphs (4 hours)	Use: • translation of $y = f(x)$ parallel to the y axis to give $y = f(x) + a$ • translation of $y = f(x)$ parallel to the x axis to give $y = f(x + a)$ • stretch of $y = f(x)$ parallel to the y axis to give $y = af(x)$ • stretch of $y = f(x)$ parallel to the x axis to give $y = f(ax)$	**Sea Defence Wall (**assignment) Two versions of an assignment in which students find functions to model the outline of a sea defence wall. The first version encourages students to work independently, the second is more structured for less able students.	

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Trigonometric Functions (8 hours)	Draw graphs of • $y = A\sin(mx + c)$ • $y = A\cos(mx + c)$ Learn the general shape and position of trigonometric functions and use the terms amplitude, frequency, wavelength, period and phase shift correctly. Fit trigonometric functions to real data. Solve trigonometric equations of the form $A\sin(mx + c) = k$ and $A\cos(mx + c) = k$	**Coughs and Sneezes (**assignment) Includes data about the way in which an outbreak of the common cold spreads. Students are asked to model the data using trigonometric and polynomial functions. **SARS A and B** (assignments) Data set giving the number of deaths from SARS. Students choose, draw and evaluate functions to model the data. **Sunrise & Sunset Times** (assignment) Students find and evaluate trigonometric functions to model how the amount of daylight varies with the day of the year. Includes data for Adelaide, Brisbane and London. **Tides** (assignment) Data set giving the water depth each hour during a day. Students choose, draw and evaluate functions to model the data.	
Linearising Data (6 hours)	Determine parameters of non-linear laws by plotting appropriate linear graphs, for example: • $y = ax^2 + b$ by plotting y against x^2 • $y = \frac{a}{x} + b$ by plotting y against $\frac{1}{x}$ • $y = ax^3 + b$ by plotting y against x^3 • $y = a\sin(x) + b$ by plotting y against $\sin(x)$ • $y = ax^b$ and $y = a^x$ using base 10 or natural logarithms	**Log Graphs** (starter) Examples (involving earthquakes and planetary motion) that can be used to introduce log graphs. Ideas of experiments and other situations that can be used for portfolio work. **Gas Guzzlers** (skills activity) Powerpoint presentation and activity in which students use a log graph to find an exponential function to model real data. **Smoke Strata** (assignment) Includes data about the height of smoke layers due to a fire in a tall building and sample examination question. Data could also be used to give practice for portfolio requirements or form the basis for an assignment.	2 Report of a piece of work in which a function is fitted to non-linear data using logarithmic or other techniques to determine parameters by plotting a linear function. Include clear explanation of method, correct algebraic notation and clearly labelled, accurate graphs. (4 hours)
Revision (6 hours)